MITRE

NOTE

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From: J. B. Hammer

Subject: A Simple Kalman Filter for Smoothing Barometric Altitude

1.0 Kalman Filter

The Kalman filter suggested for smoothing barometric height consists of a simple, two state Kalman filter. The filter's state consists of position and velocity.

Linear dynamics are assumed. The filter takes as input the measured altitude, the previous track state, and the previous track state covariance. The filter also requires as input the time of the measurement. The filter produces as output an updated track state and covariance.

2.0 Filter State and Covariance

The filter maintains a state consisting of position and velocity and a state covariance matrix which represents the uncertainty in the state estimate.
The filter state estimate in the altitude dimension is represented by the vector $\begin{bmatrix} z & \dot{z} \end{bmatrix}$. The covariance matrix of the state estimate in the x-dimension is represented by the matrix:

$$\begin{bmatrix} \sigma_z^2 & \sigma_{z\dot{z}} \\ \sigma_{z\dot{z}} & \sigma_{\dot{z}}^2 \end{bmatrix}$$

where

σ_z^2 represents the variance in the position estimate;
$\sigma_{\dot{z}}^2$ represents the variance in the velocity estimate;
$\sigma_{z\dot{z}}^2$ represents the covariance of position and velocity.

3.0 Initial State and Covariance

Before any measurements are received, the state estimates and the state covariance matrix are initialized. The state estimate is initialized by setting both the position and velocity estimate to zero. The state covariance matrix is initialized according to equation 1:

$$\begin{bmatrix} \sigma_z^2 & \sigma_{z\dot{z}} \\ \sigma_{z\dot{z}} & \sigma_{\dot{z}}^2 \end{bmatrix} = \begin{bmatrix} \infty & 0 \\ 0 & \infty \end{bmatrix} \tag{1}$$

For the purposes of an actual implementation, infinity can be represented by a large real number.

4.0 Kalman Filter Steady State Algorithm

The standard Kalman filter consists of five steps:

1. State Extrapolation
2. Covariance matrix extrapolation
3. Residual Calculation
4. State vector update
5. Covariance matrix update.

The first two steps are identical regardless of whether the measurement is a position, velocity, or both. The last three steps depend on the form of the measurement.

4.1 State Extrapolation

The first step in the filter is to extrapolate the state estimates to the time of the current measurement. Equations 2 detail the mechanics of the state extrapolation:

$$\begin{aligned} z^- &= z + \dot{z}(dt) \\ \dot{z}^- &= \dot{z} \end{aligned} \tag{2}$$

where

z^- is the predicted (extrapolated) barometric altitude;
$\dot{z}^-$ is the predicted barometric altitude rate;
z is the current barometric altitude estimate;
$\dot{z}$ is the current barometric altitude rate estimate;
dt is the time difference between the current measurement and the last state update.

4.2 Covariance Matrix Extrapolation

The next step in the filtering process is the extrapolation of the covariance matrix.

$$\sigma_{\bar z}^2 = \sigma_{\hat z}^2 + (dt)^2\sigma_{\hat{\dot z}}^2 + 2dt\,\sigma_{\hat z\hat{\dot z}} + \frac{Q(dt)^4}{4} \tag{3}$$

$$\sigma_{\bar{\dot z}}^2 = \sigma_{\hat{\dot z}}^2 + (dt)^2 Q \tag{4}$$

$$\sigma_{\bar z\bar{\dot z}} = \sigma_{\bar{\dot z}\bar z} = \sigma_{\hat z\hat{\dot z}} + (dt)\sigma_{\hat{\dot z}}^2 + \frac{(dt)^3 Q}{2} \tag{5}$$

where

Q is the process (plant) noise variance. The ratio of the plant noise to the measurement noise determines the time constant of the Kalman filter. A recommended value for Q is $(1.61)^2 \frac{ft^2}{s^4}$ for steady state tracking and $(8.05)^2 \frac{ft^2}{s^4}$ during maneuvers (see maneuver detection below).

4.3 Residual Variance

The residual (or innovations) variance is calculated as explained by equation (6):

$$\sigma_\nu^2 = \sigma_{\bar z}^2 + \sigma_{z_m}^2 \tag{6}$$

where

$\sigma_{z_m}^2$ is the variance of the barometric altitude measurement. The recommended nominal value for this parameter for barometric altimeters is 36 ft^2.

4.4 Maneuver Detection

The residual variance is used to detect maneuvers. The track residual (ν) consists of the difference between the new measurement and the measurement prediction (from equation 2):

$$\nu = (z_m - \bar z) \tag{7}$$

A maneuver is detected when the residual is greater than a threshold based on the residual variance, i.e., if:

$$\nu^2 > k\sigma_\nu^2 \tag{8}$$

(where k is a unit-less constant -- an adaptation parameter with a recommended value of 9), then a maneuver is declared.

If a maneuver is detected the covariance matrix extrapolation is recalculated as per section 4.2 with the higher value of Q, or process noise, assumed. The higher value for Q results in a higher gain, and less measurement smoothing. This is appropriate during a maneuver. After recalculating the predicted covariance matrix, computation resumes as described in the steps below.

4.5 Filter Gain

The gain vector (**w**) is then calculated according to equations (9) and (10):

$$w_0 = \frac{\sigma_{\tilde{z}}^2}{\sigma_v^2} \tag{9}$$

$$w_1 = \frac{\sigma_{\tilde{z}\tilde{\dot{z}}}}{\sigma_v^2} \tag{10}$$

4.6 State Estimate Smoothing (Update)

The update of the state estimate is performed according to equations (11) and (12):

$$\hat{z} = \tilde{z} + w_0 (z_m - \tilde{z}) \tag{11}$$

$$\hat{\dot{z}} = \tilde{\dot{z}} + w_1 (z_m - \tilde{z}) \tag{12}$$

where

z_m is the current position measurement.

4.7 Covariance Matrix Update

The update of the covariance matrix is then performed according to equations 13, 14, and 15:

$$\sigma_{\hat{z}}^2 = (1 - w_0)\, \sigma_{\tilde{z}}^2 \tag{13}$$

$$\sigma_{\hat{z}\hat{\dot{z}}} = (1 - w_0)\, \sigma_{\tilde{z}\tilde{\dot{z}}} \tag{14}$$

$$\sigma_{\hat{\dot{z}}}^2 = \sigma_{\tilde{\dot{z}}}^2 - w_1 \sigma_{\tilde{z}\tilde{\dot{z}}} \tag{15}$$

The update of the state and covariance completes the Kalman filter operations for the current measurement.

5.0 Conclusions

A formulation for a simple, two state, Kalman filter which may be practical for smoothing barometric altitude measurements has been presented. The filter accepts as input barometric altitude measurements.

Jonathan B. Hammer
Principal Engineer
MITRE Corporation
McLean, Virginia