RTCA Special Committee 186, Working Group 5

ADS-B UAT MOPS

Meeting #3

Hard Decision versus Erasure Decoding

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SUMMARY

In this paper we compare hard decision decoding to erasure decoding for each of the three UAT messages recommended as UAT enhancements. Although erasures may improve message error performance they require adding additional CRC symbols to meet message error detection requirements.

There has been some discussion about employing demodulation circuitry that declares binary erasures. If n[1] or more binary erasures occur in an 8-bit symbol, that symbol would be declared an erasure. The input to the Reed Solomon decoder would be 256'ary symbols and erasures. It is generally known that using erasures in the decoding process increases the probability of an undetected word error compared to hard decision decoding. The purpose of this paper is to quantify this increase for the three UAT Reed Solomon message formats recommended by Wilson and Leiter as design enhancements.

The UAT specification specifies a 10^{-6} upper bound on the probability of an undetected message error. This bound can be met by a combination of Reed Solomon decode failure and CRC coding. Clearly if decode failure probability cannot meet the 10^{-6} upper bound, then CRC coding must be added.

The probability of a Reed Solomon word error and the probability of an undetected word error, for both hard decision decoding and decoding with erasures, are given in the appendix.

To determine the upper bound on undetected error for hard decision decoding, we varied the symbol error probability to maximize the probability of an undetected message error. However, for erasure decoding, we independently varied symbol error probability and erasure probability. This does gives a worse case upper bound, but one could argue that a selected demodulation scheme may prohibit the erasure and error probability combination that gave the worse case bound. Since there is no proposed scheme we will use the worse case bound.

Table 1 addresses decode failure probability for the two one word ADS-B messages. With hard decisions the upper bound on undetected error probability occurs at a symbol error probability equal to 1. The (46,34) message meets the bound requirement but the (26,18) message just misses. With erasures, the upper bound for both messages is nowhere close to the bound. For both messages the upper bound occurs when the average number of erasures per message equals the maximum number that the code can correct and when there are no correct symbols into the decoder.

Table 2 addresses decode failure for the 6 word uplink message. The probability of an undetected message error, P_{um}, is equal to the probability of no detected word errors ($1-P_{dwe}$) in each of the six words minus the probability of no errors ($1-P_{we}$) in each of the six words. Therefore

$$P_{um} = (1-P_{dwe})^6 - (1-P_{we})^6$$

The probability of a detected word error is equal to the probability of a word error minus the probability of an undetected word error P_{uwe}. Therefore

[1] One or perhaps two would appear to be reasonable choice.

$$P_{dwe} = P_{we} - P_{uw}$$

Substituting

$$P_{um} = (1-P_{we}+P_{uwe})^6 - (1-P_{we})^6$$

In this case, we will vary symbol error probability (and, when appropriate, symbol erasure probability) to maximize P_{um}. Table 2 shows the results for hard decisions and Table 3 shows the results when erasures are present. The uplink message meets the 10^{-6} undetected message error requirement with hard decisions but falls short by more than three orders of magnitude when erasures can be present.

ADS-B Message	**Hard Decision Decoding**		**Erasure Decoding**		
	Upper Bound on Undetected Error Probability	Symbol Error Probability	**Upper Bound on Undetected Error Probability**	Symbol Erasure Probability	Symbol Error Probability
(46,34) RS	$3.25\ 10^{-8}$	1	0.15	8/46	38/46
(26,18) RS	$3.43\ 10^{-6}$	1	0.18	12/26	14/26

Table 1. Bounds on ADS-B Undetected Error Probabilities

6x(85,65) Uplink Message	**Upper Bound on Undetected Message Error Probability**	Undetected Word Error Probability	Word Error Probability	Symbol Error Probability
	$5.51\ 10^{-13}$	$2.64\ 10^{-13}$	0.19	0.096

Table 2. Bounds on Uplink Undetected Error Probability for Hard Decision Decoding

6x(85,65) Uplink Message	**Upper Bound on Undetected Message Error Probability**	Undetected Word Error Probability	Word Error Probability	Symbol Error Probability	Symbol Erasure Probability
	$6.9\ 10^{-3}$	$4.4\ 10^{-2}$	0.54	20/85	0.06

Table 3. Bounds on Uplink Undetected Error Probability for Erasure Decoding

In summary, with hard decision decoding, decode failure rate is sufficient to meet the 10^{-6} requirement for all but the (26,18) ADS-B message. There, 1 CRC symbol[2] is more than sufficient to have the combination of decode failure and CRC coding satisfy the requirement. With erasure decoding none of the messages meet the 10^{-6} requirement. The ADS-B messages require 3 CRC symbols and the uplink message requires 2 CRC symbols.

Appendix

The equations given here were taken from a 1975 Hughes Aircraft Company subsystem design document for the JTIDS program.

(n,k) Reed Solomon code

q = alphabet size(e.g., 256 in this paper)

d = n-k+1 minimum distance

and define xmax = -1 and emax = IntegerPart[d/2]

pe = probability of a symbol error, px=the probability of a symbol erasure

N[w] = number of code words of weight w

$$N[w] = \binom{n}{w} \sum_{h=0}^{w-1-(n-k)} (-1)^h \binom{w}{h} \left(q^{w-h-(n-k)} - 1\right)$$

For hard decision decoding

$$P_{uwe} = \sum_{w=d}^{n} N[w] \sum_{lw=0}^{e\max} pp$$

$$pp = \sum_{j=0}^{lw} \sum_{i=0}^{lw-j} \sum_{k=0}^{lw-i-j} Multinomial[w-k-j,k,j](q-2)^j \binom{n-w}{i} (q-1)^i \left(\frac{p_e}{q-1}\right)^{w-k+1} (1-p_e)^{n-w+k-i}$$

and

$$P_{we} = 1 - \sum_{i=o}^{e\max} \binom{n}{i} p_e^i (1-p_e)^{n-i}$$

[2] We must select an integer number of 8-bit symbols.

For erasure decoding

$$P_{uwe} = \sum_{w=d}^{n} N[w] \sum_{lw=0}^{e\max} \sum_{r=0}^{x\max-2lw} pp$$

$$pp = \sum_{j=0}^{lw} \sum_{i=0}^{lw-j} \sum_{k=0}^{lw-i-j} \sum_{t=0}^{r} \sum_{s=0}^{r-t} Multinomial[w-k-j-t,k,j,t](q-2)^{j}$$

$$Multinomial[n-w-i-s,i,s](q-1)^{i}\left(\frac{p_e}{q-1}\right)^{w-k-t+1} p_x^{t+s}(1-p_e-p_x)^{n-w+k-i-s}$$

and

$$P_{we} = 1-\sum_{ke=0}^{e\max} \sum_{ku=0}^{xma-2ke} Multinomial[ke,ku,n-ke-ku]p_e^{ke} p_x^{ku}(1-p_e-p_x)^{n-ke-ku}$$