RTCA Special Committee 186, Working Group 5

ADS-B UAT MOPS

Meeting #10

UAT Error Detection and Correction Performance

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SUMMARY

This paper proposes a possible appendix for the UAT MOPS. It provides a brief synopsis of the error detection and correction (EDAC) performance of the Reed Solomon coding used for the various message types. It is proposed that this appendix should be referenced in the appropriate sections of 2.2 to indicate both the levels of undetected message error rate performance provided and why no specific requirement or test for false message reception rate is required.

This appendix provides information on the performance of the Reed Solomon (RS) codes used by the various burst types of UAT. The basic ADS-B message uses a RS(30,18) code; the long ADS-B message uses a RS(48,34) code; and the ground up link message uses a RS(92,72) code six times. These codes provide very strong error correction. Also, the error detection provided by these codes is sufficient to provide a maximum undetected error rate that is less than 10^{-8} for each of the burst types, so additional CRC coding is not needed.

The total word error rate for a RS(n,k) code is given by the formula

$$P_E = \sum_{j=t+1}^{n} \frac{n!}{j!(n-j)!} p^j (1-p)^{n-j}$$

where t=(n-k)/2 and p is the channel bit error rate (BER). P_E includes both undetected and detected word error probabilities.

The asymptotic value for the undetected word error rate (achieved when the channel bit error rate is 0.5) for a RS(n,k) code can be calculated using the formula

$$P_U = \frac{256^k - 1}{256^n} \sum_{j=0}^{t} \frac{n!}{j!(n-j)!} 255^j$$

where t=(n-k)/2. The results are given in Table Z-1.

Code	Maximum Undetected Word Error Rate
RS(30,18)	2.06e-9
RS(48,34)	9.95e-10
RS(92,72)	5.74e-12

Table Z-1. Maximum Undetected RS Word Error Rates

The undetected error performance of a RS code as a function of channel bit error rate can also be calculated, but the mathematical complexity is much greater [1]. The results are shown in figures Z-1 through Z-3. These graphs show total word error rate together with undetected word error rate. The detected word error rate, P_D, is just the difference between the two curves. If the correct word error rate is defined as P_C, then all the probabilities are related by

$$1 = P_E + P_C = P_U + P_D + P_C .$$

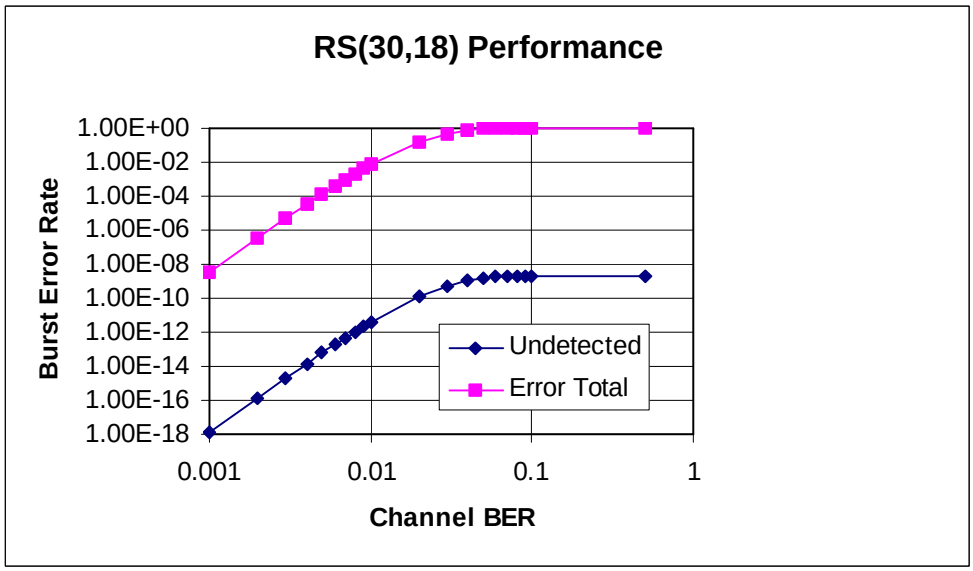

Figure Z-1. Basic ADS-B Message Performance
("Undetected" = P_U; "Error Total" = P_E.)

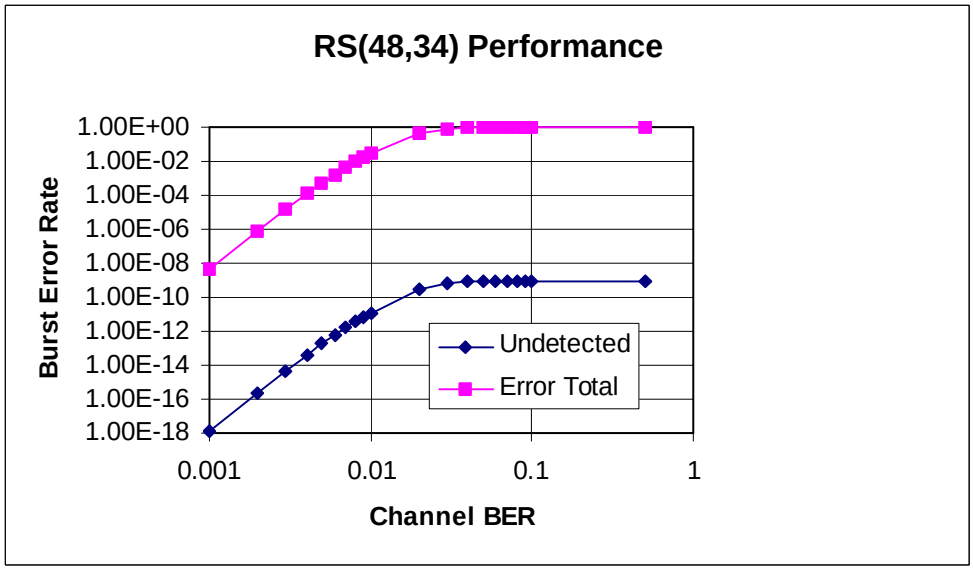

Figure Z-2. Long ADS-B Message Performance
("Undetected" = P_U; "Error Total" = P_E.)

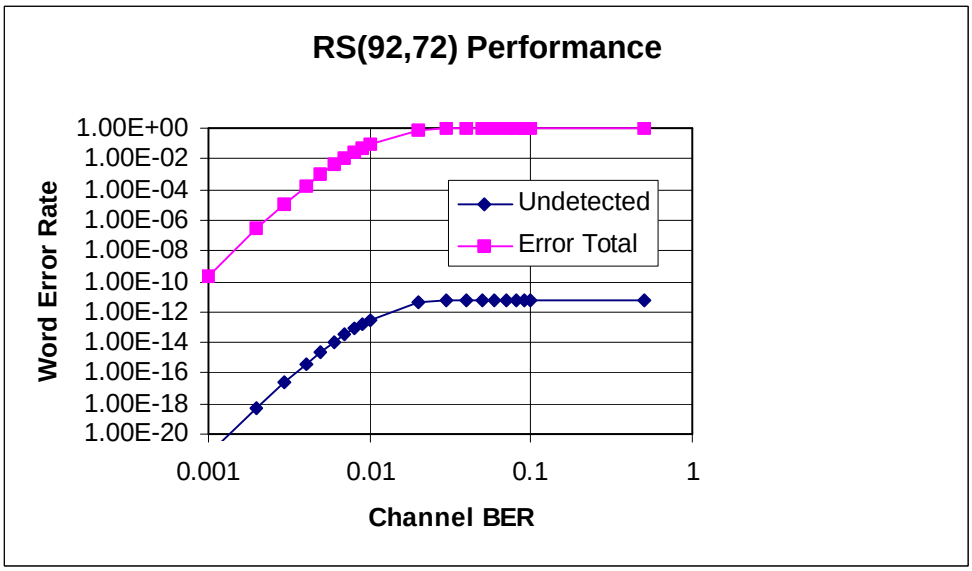

Figure Z-3. Ground Up Link Message Performance
("Undetected" = P_U; "Error Total" = P_E.)

Note that for the ADS-B messages, the word error rate is equal to the burst error rate because there is one word per burst. This is not true for the ground up link message. Figure Z-3 shows the performance of a single RS(92,72) word. The performance of an entire burst, consisting of six words, is given by

$$P_{Uburst} = (1 - P_E + P_U)^6 - (1 - P_E)^6 = (P_C + P_U)^6 - P_C{}^6$$

and

$$P_{Eburst} = 1 - (1 - P_E)^6 = 1 - P_C{}^6$$

Again, P_E is the total word error rate, and P_U is the undetected word error rate. A graph of the undetected burst error rate versus the channel BER is shown in Figure Z-4. This figure indicates that the maximum undetected error rate is about 1.3e-12, which occurs when the channel BER is about 0.012. To see why there is a maximum, consider the following approximation:

$$P_{UBurst} \approx 6P_U(1 - P_E)^5.$$

The P_U term is small at low BER and the $(1-P_E)^5$ term is small at high BER (because P_E is nearly 1 in that case).

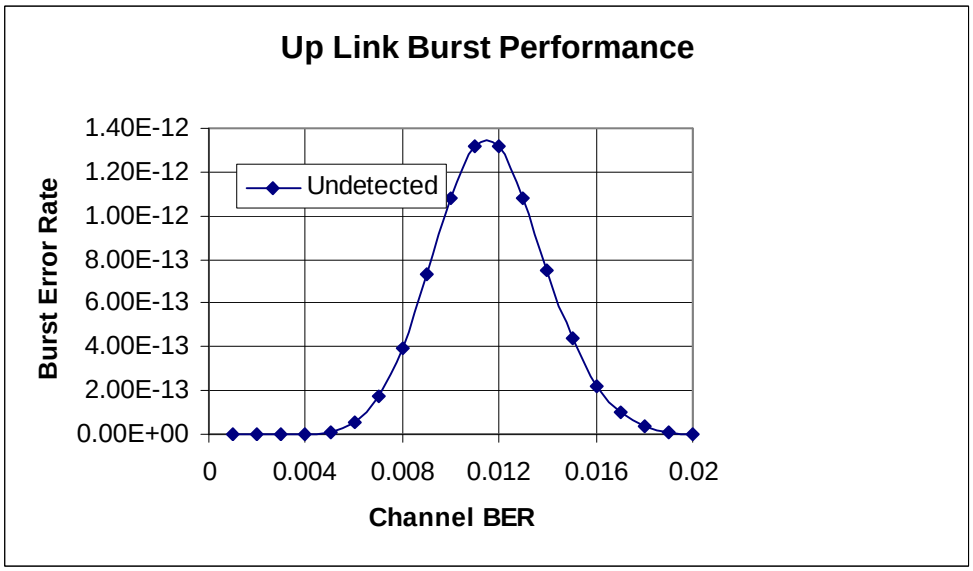

Figure Z-4. Ground Up Link Message Undetected Burst Error Rate
("Undetected" = P_{UBurst} .)

Reference [1]: Kasami,T., and S. Lin, 1984, "On the Probability of Undetected Error for Maximum Distance Separable Codes," IEEE Trans. Comm., COM-32,998-1006.