GPS Integrity Monitoring and Associated ADS-B Operational Characteristics

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GPS integrity monitoring relationships for 2-DOF Chi-sq model (6 or more satellites in view)

Normalized χ^2 model for 2-DOF:

Central Chi-sq distribution for 2-DOF:Test against sum of squares of mean errors, x

$$fc2(x) := \frac{1}{2}\cdot \exp\left(\frac{-x}{2}\right) \qquad a := 27.6 \qquad TDn := \sqrt{a} \qquad TDn = 5.254$$

$$Pfa(a) := \int_{a}^{\infty} \frac{1}{2}\cdot \exp\left(\frac{-x}{2}\right) dx \qquad Pfa(a) = 1.016\times 10^{-6}$$

Note: Assumes decorrelation time of 6 min. Resulting false alert rate is 10^-5/hr

Noncentral Chi-sq distribution for 2-DOF and noncentrality parameter, λ:

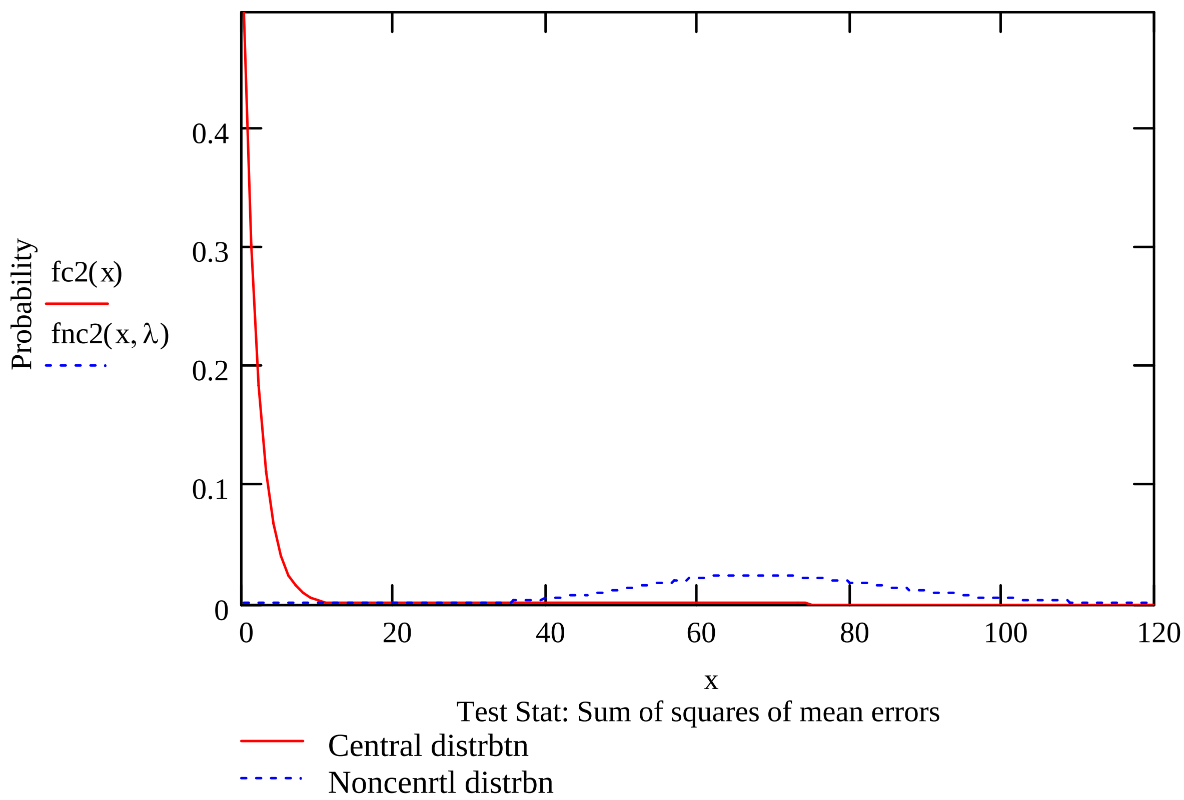
$$fnc2(x,\lambda) := \frac{1}{2}\cdot \exp\left[-\left(\frac{x+\lambda}{2}\right)\right]\cdot \sum_{j=0}^{50} \left(\frac{\lambda^{j}\cdot x^{j}}{2^{2\cdot j}\cdot j!\cdot j!}\right) \qquad \lambda := 68.3 \qquad pbiasn := \sqrt{\lambda} \qquad pbiasn = 8.264$$

$$Pmd(\lambda) := \int_{0}^{a} \frac{1}{2}\cdot \exp\left[-\left(\frac{x+\lambda}{2}\right)\right]\cdot \sum_{j=0}^{50} \left(\frac{\lambda^{j}\cdot x^{j}}{2^{2\cdot j}\cdot j!\cdot j!}\right) dx \qquad Pmd(\lambda) = 1.013\times 10^{-3}$$

$$x := 0, 1 .. 120$$

Chi-sq distributions vs test statistic (sum of squares of normalized mean errors)

$a = 27.6 \quad TDn = 5.254 \quad Pfa(a) = 1.016\times 10^{-6} \quad \lambda = 68.3 \quad pbiasn = 8.264 \quad Pmd(\lambda) = 1.013\times 10^{-3}$



Normalized Chi square distributions for 2-DOF and above integrity monitoring parameters

Example showing Thd setting for desired Pfa and resulting fault error bound, Ro, at required Pmd

GPS integrity monitoring example:

Normalized threshold for fault detection: $a := 27.6$ $TDn = 5.254$ $Pfa(a) = 1.016\times 10^{-6}$

For assumed GPS horizontal plane stdr dev, σ meters: $\sigma := 10$ $EPU := 2\cdot\sigma$ $EPU = 20$

Threshold setting for pfa: $TD := TDn\cdot\sigma$ $TD = 52.536$

Normalized detected error with prob Pmd: $\lambda o := 68.3$ $Pmd(\lambda o) = 1.013\times 10^{-3}$ $pbiaso := \sqrt{\lambda o}$

Protected radius, Ro, for assumed max satellite slope factor, sm, (1< sm < 4): $sm := 2$

$Ro := pbiaso\cdot\sigma\cdot sm$ $Ro = 165.288$ $\frac{Ro}{\sigma} = 16.529$

For GPS fault rate 10^-4/hr, HPL at risk level, Ir: $Iro := 10^{-4}\cdot Pmd(\lambda o)$ $Iro = 1.013\times 10^{-7}$

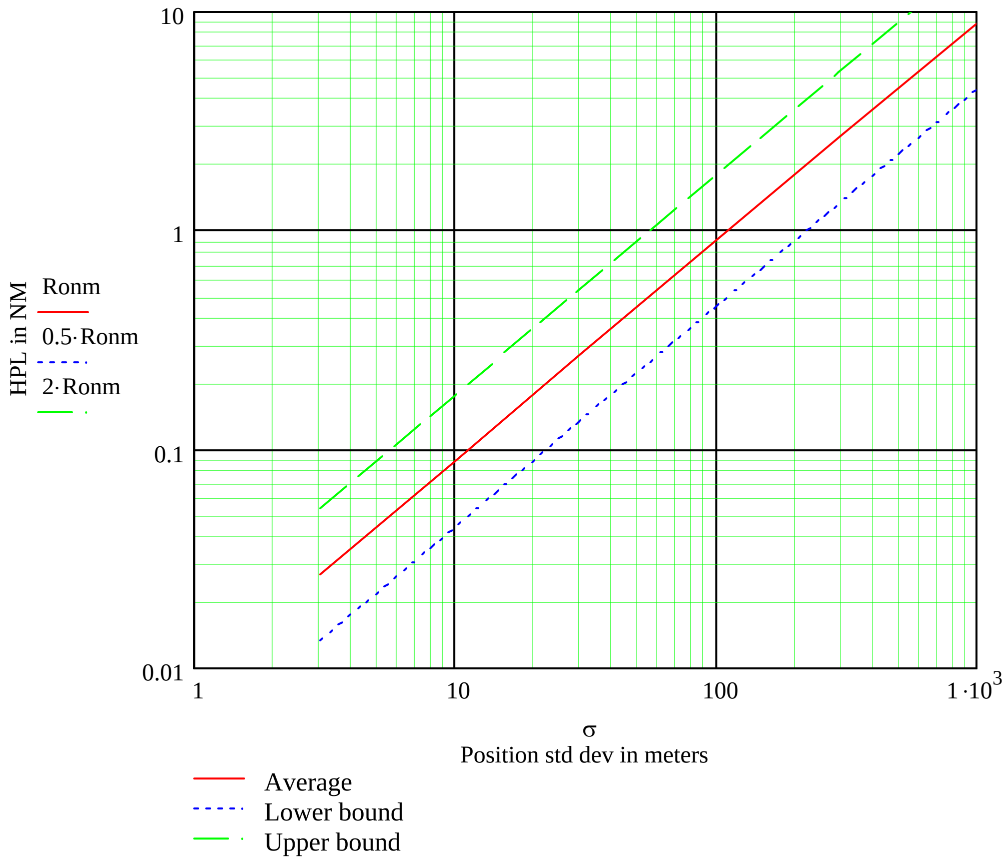
HPL in NM is Rc: $Rc := \frac{Ro}{1852}$ $Rc = 0.089$ NM

Note that the range of Ro is between one half and twice this value for the four-to-one sm variation

ADS-B encoding of these values for NACp => EPU and NIC => Ro:

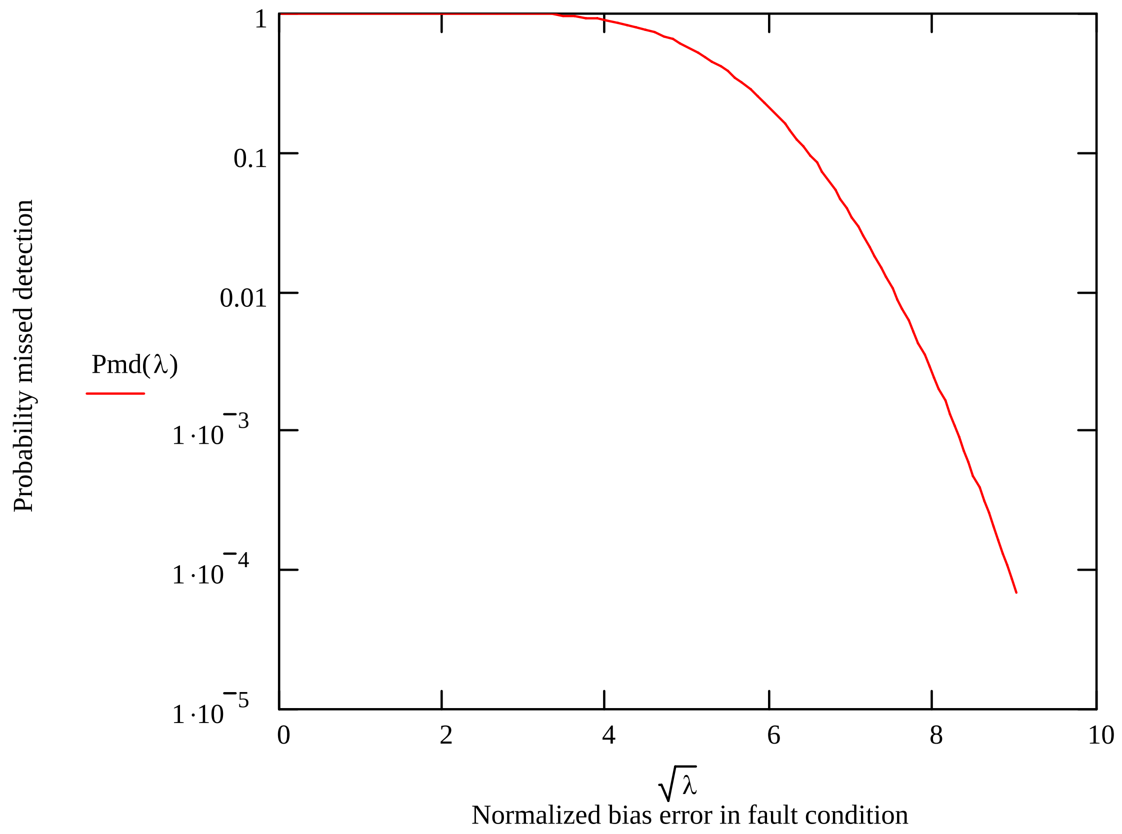
- NACp = 9, or 30 m, (assuming VEPU < 45 m for this NACp requirement)
- NIC = 8, or 185.2 m, with range of 8 (for sm = 1) to 7, or 370.4 m, (for sm = 4)
- Associated risk level for NIC is Iro = 10^-7/hr

Dependence of HPL, Ro at Ir = 10^-7/hr, on horizontal position std dev and failed satellite slope factor



Variation of Pmd with normalized fault condition pbias error for desired Thd setting

$a = 27.6 \quad TDn = 5.254 \quad pbiasn = 8.264 \quad Pfa(a) = 1.016 \times 10^{-6}$



Integrity monitoring variation vs normalized horizontal position error in fault condition

Illustration of determination of containment radius at lower integrity risk level, Ir = 10^-5/hr

For fault detection containment radius at other lower risk levels, say 10^-5/hr, Pmd = 0.1 and:

Reference baseline at 10^-7/hr: $\lambda o := 68.3 \quad Pmd(\lambda o) = 1.013 \times 10^{-3} \quad pbiaso := \sqrt{\lambda o} \quad pbiaso = 8.264$

For Pmd = 0.1: $\lambda := 41.4 \quad Pmd(\lambda) = 0.103 \quad pbiasp := \sqrt{\lambda} \quad pbiasp = 6.434$

Reduction factor for containment radius at Ir = 10^-5/hr is then γ: $\gamma := \frac{pbiasp}{pbiaso} \quad \gamma = 0.779$

$Rp := \gamma \cdot Ro \quad Rp = 128.686 \quad Ir := 10^{-4} \cdot Pmd(\lambda) \quad Ir = 1.026 \times 10^{-5}$

Example of relationship between GPS integrity monitoring parameters and ADS-B/GPS operational features

For GPS failure rate, qr/hr, and assumed decorrelation time interval, ta min:

$qr := 10^{-4}$ $\quad ta := 6$ $\quad m := \frac{60}{ta}$ $\quad m = 10$

ADS-B integrity risk, I, that GPS error is greater than Ro w/o alert during a close approach time te = 30 min:

$te := 0.5$ $\quad \lambda := 68.3$ $\quad Pmd(\lambda) = 1.013 \times 10^{-3}$ $\quad I := Pmd(\lambda) \cdot \frac{qr}{te}$ $\quad I = 2.026 \times 10^{-7}$

False alert rate per hr, FAR, for decorrelation interval, ta min:

$a := 27.6$ $\quad Pfa(a) = 1.016 \times 10^{-6}$ $\quad FAR := Pfa(a) \cdot m$ $\quad FAR = 1.016 \times 10^{-5}$

Probability of service loss, q, assuming failed satellites are replaced except for a fraction, fe, of detected errors that are not excluded (Hourly rate for service continuity estimate):

$fe := 10^{-3}$ $\quad qe := (1 - Pmd(\lambda)) \cdot fe$ $\quad q := qr \cdot qe$ $\quad q = 9.99 \times 10^{-8}$

GPS loss of function rate (continuity), Cn/hrs: $Cn := q + FAR$ $\quad Cn = 1.026 \times 10^{-5}$

Joint behavior of GPS continuity of service and ADS-B/GPS avionics MTBF

ADS-B/GPS avionics reliability over time, T, for equipment MTBF = Ms hrs: $Ms := 20000$

Single string avionics reliability, R: $R(T, Ms) := \exp\left(\frac{-T}{Ms}\right)$

Redundant hot standby avionics reliability, A: $A(T, Ms) := 1 - (1 - R(T, Ms))^2$

Resulting ADS-B/GPS continuity of service over T hrs for single and dual avionics equipage:

$$Rts(T, q, FAR, Ms) := [1 - (q + FAR) \cdot T] \cdot R(T, Ms)$$

$$Rtp(T, q, FAR, Ms) := [1 - (q + FAR) \cdot T] \cdot A(T, Ms)$$

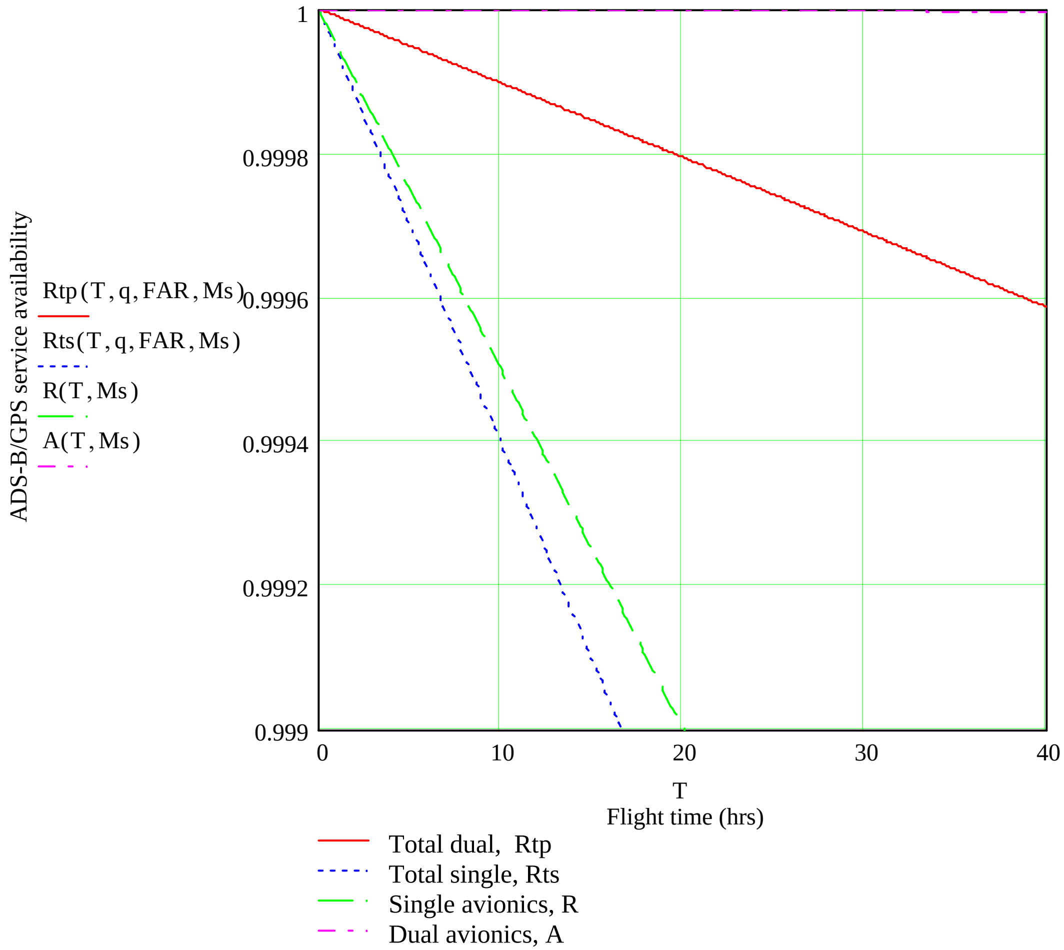
Resulting rate of loss of ADS-B/GPS continuity of service (per hr) for single and dual equipage:

$$Crs := 1 - Rts(1, q, FAR, Ms) \qquad Crs = 6.025 \times 10^{-5}$$

$$Crp := 1 - Rtp(1, q, FAR, Ms) \qquad Crp = 1.026 \times 10^{-5}$$

$\sigma = 10$ $\quad Ro = 165.288$ $\quad FAR = 1.016 \times 10^{-5}$ $\quad I = 2.026 \times 10^{-7}$ $\quad q = 9.99 \times 10^{-8}$

$ta = 6$ $\quad m = 10$ $\quad Ms = 2 \times 10^{4}$ $\quad Crs = 6.025 \times 10^{-5}$ $\quad Crp = 1.026 \times 10^{-5}$



ADS-B/GPS service reliability for single and dual equipage as a function of flight time in hours for above system parameters. Avionics only reliability also shown for reference.