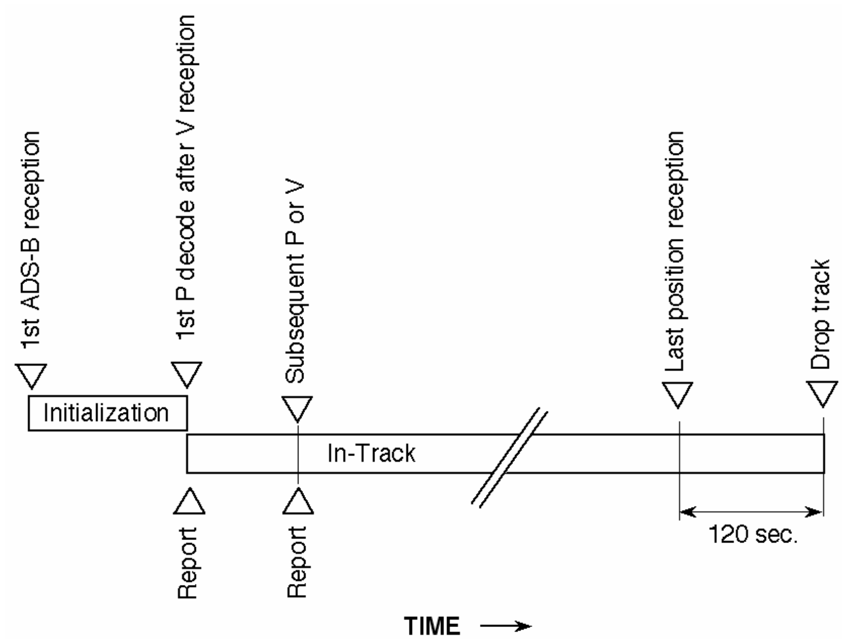
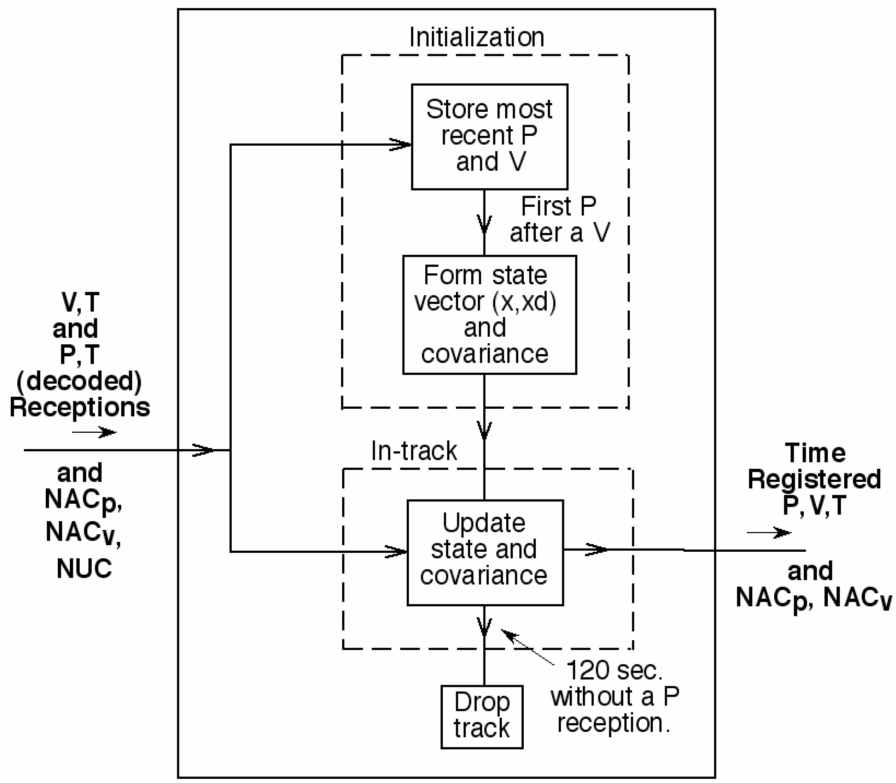
1.1.1.1.1.1.1.1 Time Registered Position and Velocity Calculation

As each 1090ES Airborne Position Message or Airborne Velocity Message (Subtype 1 or 2) is received, a calculation is performed to update the State Vector, which expresses the time registered position and velocity of the target. The calculation, which is a form of Kalman filter, **shall** be performed as defined in the following subsections. Figure 3.2.3.2.1.1.2.3-1 and Figure 3.2.3.2.1.1.2.3-2 provide guidance material to illustrate this process.



<u>Figure 3.2.3.2.1.1.2.3-1:</u> Timeline Illustration of the Time Registration Process



<u>Figure 3.2.3.2.1.1.2.3-2:</u> The Time Registration Process

1.1.1.1.1.1.1.1.1 Association by 24 bit Address

The different targets being received interleaved in time are separated according to their 24-bit Addresses and the Address Qualifier that indicates whether the address is an ICAO 24-bit Address or an anonymous 24-bit Address (see Table 3.2.3.2.1.1.1.1). Receptions having different addresses **shall** be treated separately.

1.1.1.1.1.1.1.1.2 Position and Velocity Components

a. In the calculation specified here, position is defined by the following three components,

x = longitude in degrees
y = latitude in degrees
z = altitude in feet

Each of these is to be treated separately in the calculation.

b. Velocity is defined by the corresponding three components, which are the time derivatives of the position components.

   xd = rate of change of longitude (degrees per second)
   yd = rate of change of latitude (degrees per second)
   zd = rate of change of altitude (feet per second)

1.1.1.1.1.1.1.1.3 State and Covariance

In each component, the calculation maintains a state consisting of position and rate and a covariance matrix which indicates the uncertainty in the current values. After a track is started, the state and covariance matrix **shall** be stored from one update to the next until the track is dropped.

The position and velocity state in the x-dimension is represented by the vector: (x,xd). In the following formulas, the variable "x" is used to show the filter state in one of the three dimensions; the y and z dimensions have a state and covariance representation which exactly parallels that of x. The covariance matrix of the state estimate in the x-dimension is represented by the matrix:

$$\begin{matrix} Sig^2x & Sigxxd \\ Sigxxd & Sig^2xd \end{matrix}$$

where

Sig^2x = the variance of x;

Sig^2xd = the variance of xd;

Sigxxd = the covariance of x and xd.

1.1.1.1.1.1.1.1.4 Accuracy Inputs to the Registration Process

The inputs to the Registration Process include position and velocity, and also information about accuracy (as stated in §**Error! Reference source not found.**). As shown in Table 3.2.3.2.1.1.1.1, some of the accuracy information arrives in messages other than position and velocity messages. Therefore the most recently received accuracy information is stored for use as input to this process. Specifically;

a. the most recent value of NAC_P **shall** be stored for use as an input, and the same is true for NAC_V information.

b. for aircraft that are transmitting NUC, the most recent values of NUC_P and NUC_R **shall** be stored for use as inputs.

1.1.1.1.1.1.1.1.5 Quantization

a. In order to assure that quantization effects in these calculations do not degrade performance, the position quantization step size **shall** be ≤0.2 meter for each component.

b. The velocity quantization step size **shall** not exceed 0.01 meter per second for each component.

Note: *Although these maximum step sizes are given here in meters and meters per second, they must be expressed in the same units as latitude, longitude, and altitude. Appropriate conversion factors are given in §1.1.1.1.1.1.1.1.6 and §1.1.1.1.1.1.1.1.9.*

1.1.1.1.1.1.1.1.6 Initialization

a. For any particular 24-bit Address plus Address Qualifier, the tracking process begins by initializing the state and the covariance matrix. Initialization **shall** be performed at the first time when a position has been received and calculated (see §**Error! Reference source not found.**) after having received a Velocity Message. Target State is initialized by setting:

x = the received value of x
xd = the received value of xd

b. The preceding paragraph applies when the first receptions from this target are in the airborne format (in which position and velocity are in separate messages). Targets being received in the surface format are not processed for position-velocity registration because each reception is already in that form. When a target that was transmitting in the surface format changes to the airborne format, the position-velocity registration process **shall** begin at the time of the first reception in the airborne format. If the first such reception is a position message, target state **shall** be initialized by setting:

x = the received value of x
xd = the most recent velocity reception

c. If the first such reception is a Velocity Message, the Target State **shall** be initialized by using the most recent surface reception for both position and velocity. The new velocity reception **shall** be processed as an update of the track.

d. The covariance matrix **shall** be initialized according to whether NAC_P and NAC_V or NUC_P and NUC_R information has been received from this target. In all cases, the cross terms are initially set to zero:

Sigxxd = Sigyyd = Sigzzd = 0

e. If NAC_P has been received, the position elements **shall** be initialized as follows.

$$Sig^2x = (Sig(x) \text{ from Table } 3.2.3.2.1.1.2.3.6A)^2$$

where the standard deviation values for x, y, and z are obtained from Table 3.2.3.2.1.1.2.3.6A.

Table 3.2.3.2.1.1.2.3.6A: Conversion from NAC_P to Sig(x), Sig(y), and Sig(z)

NAC_P	**Sig(x), Sig(y)** meters	**Sig(z)** feet
11	1.2	6.6
10	4.1	25
9	12	74
8	38	100
7	76	100
6	230	100
5	380	100
4	760	100
3	1500	100
2	3000	100
1	7600	100
0	20000	100

f. If NAC_V has been received, the velocity elements **shall** be initialized as follows.

$$Sig^2xd = (Sig(xd) \text{ from Table } 3.2.3.2.1.1.2.3.6B)^2$$

where the standard deviation values for xd, yd, and zd are obtained from Table 3.2.3.2.1.1.2.3.6B.

Table 3.2.3.2.1.1.2.3.6B: Conversion from NAC_V to Sig(xd), Sig(yd), and Sig(zd)

NAC_V	**Sig(xd), Sig(yd) m/sec.**	**Sig(zd) ft./sec.**
4	0.12	0.76
3	0.41	2.5
2	1.20	7.5
1	4.10	25
0	10	50

g. If NUC_P has been received, then the position elements **shall** be initialized as follows.

$$Sig^2x = (Sig(x) \text{ from Table 3.2.3.2.1.1.2.3.6C})^2$$

where the standard deviation values for x, y, and z are obtained from Table 3.2.3.2.1.1.2.3.6C,

<u>Table 3.2.3.2.1.1.2.3.6C:</u> Conversion from NUC_P to Sig(x), Sig(y), and Sig(z)

NUC_P	Sig(x), Sig(y) meters	Sig(z) feet
9	1.2	5.4
8	4.1	20
7	38	100
6	76	100
5	189	100
4	378	100
3	756	100
2	3782	100
1	7565	100
0	15000	100

<u>Note:</u> *The values of Sig(x) and Sig(y) in this table were obtained from the Extended Squitter MOPS, RTCA/DO-260, Table 2-67. For example, when NUC_P = 7, the corresponding entries in Table 2-67 indicate that the 95th percentile horizontal error is 0.05 NM or less. The conversion from 95th percentile error to the one-sigma value of x or y is a reduction factor of 2.45. Therefore the one-sigma value of x is 0.02 NM which equals 38 meters.*

h. If NUC_R has been received, then the velocity elements **shall** be initialized as follows.

$$Sig^2xd = (Sig(xd) \text{ from Table 3.2.3.2.1.1.2.3.6D})^2$$

<u>Table 3.2.3.2.1.1.2.3.6D:</u> Conversion from NUC_R to Sig(xd), Sig(yd), and Sig(zd)

NUC_R	Sig(xd), Sig(yd)	Sig(zd)
	m/sec.	ft./sec.
4	0.12	0.76
3	0.41	2.5
2	1.20	7.6
1	4.10	25
0	10	50

Note: *The values in this table were derived from the 1090ES ADS-B MOPS, RTCA/DO-260, Table 2-20.*

i. In the absence of NAC_P, NAC_V, or NUC_P, NUC_R information, the covariance matrix elements **shall** be initialized as follows.

$Sig^2x = Sig^2y = (20{,}000\ m)^2$

$Sig^2xd = Sig^2yd = (10\ m/sec)^2$

$Sig^2z = (100\ ft.)^2$

$Sig^2zd = (50\ ft./sec.)^2$

The above values given in meters may be converted to degrees longitude and degrees latitude by the following formulas.

(value in degrees longitude) = (value in meters)/Mx

(value in degrees latitude) = (value in meters)/My

where

My = 111112.5 meters per degree latitude

Mx = My * cos(latitude)

Note: *As specified in this section, initialization of the covariance matrix depends on whether NAC_P and NAC_V or NUC_P and NUC_R have been received, or none of these. Which of the two types of information will depend on which of the two avionics versions is transmitting (RTCA/DO-260 or RTCA/DO-260A). This distinction is initially unknown by the BSGS for each target, and must be determined from the Version Number information conveyed within 1090ES Operational Status Messages (as specified in RTCA/DO-260A, §2.2.3.2.7.2.5). It is possible that the Time Registration process will be initiated by position and velocity receptions prior to the version determination. As stated in RTCA/DO-260A §2.2.3.2.7.2.5 and RTCA/DO-260A Appendix N §N.1.2, the BSGS will initially treat receptions as if they were in Version ZERO (0) (RTCA/DO-260 compliant) 1090ES ADS-B Message formats. If this is still true at the time of initialization, the NUC_P and NUC_R specifications of this section will apply. If it is subsequently determined that Version ONE (1) applies, then the NAC_P and NAC_V values will be used in updates of the covariance matrix from that time on.*

1.1.1.1.1.1.1.1.7 State Update

After initialization, as each new measurement is received, whether position or velocity, an update **shall** be performed, yielding a new value of the state (x,xd) for the Time of Applicability of the new reception. The update calculations are defined using the following notation.

Previous state = (x(prev), xd(prev))

The previous covariance matrix is denoted,

$Sig^2x(prev)$	$Sigxxd(prev)$
$Sigxxd(prev)$	$Sig^2xd(prev)$

Extrapolation from the previous state to the new Time of Applicability is performed.

Extrapolated state = (x(extr), xd(extr))

The extrapolated covariance matrix is denoted:

$Sig^2x(extr)$	$Sigxxd(extr)$
$Sigxxd(extr)$	$Sig^2xd(extr)$

The new reception, if a position, is denoted x(meas). If a velocity, the new reception is denoted xd(meas). After the update is performed, the result is a current estimate of the state, denoted:

Updated state = (x(est), xd(est))

and a current estimate of the covariance matrix, denoted:

$Sig^2x(est)$	$Sigxxd(est)$
$Sigxxd(est)$	$Sig^2xd(est)$

1.1.1.1.1.1.1.1.8 State Extrapolation

As each new measurement is received, whether position or velocity, the state (x,xd) **shall** be extrapolated to the Time of Applicability using the following formulas.

x(extr) = x(prev) + xd(prev) * DeltaT

xd(extr) = xd(prev)

where DeltaT is the time difference in seconds between the new Time of Applicability and the previous value.

1.1.1.1.1.1.1.1.9 Covariance Matrix Extrapolation

The covariance matrix **shall** also be extrapolated to the new Time of Applicability, using the following formulas.

$Sig^2x(extr) = Sig^2x(prev) + DeltaT^2 * Sig^2xd(prev) + 2 * DeltaT * Sigxxd(prev) + DeltaT^4 * Q/4$

$Sig^2xd(extr) = Sig^2xd(prev) + DeltaT^2 * Q$

$Sigxxd(extr) = Sigxxd(prev) + DeltaT * Sig^2xd(prev) + DeltaT^3 * Q/2$

where the value of "Q" **shall** be equivalent to one times the acceleration of gravity (1G), which will be defined by the OUTLIER_ACCEL_GRAVITY configuration parameter. The nominal value of "Q" is one times the acceleration of gravity (1G, approximately $[9.75 \text{ meters/sec}^2]^2$), and its value **shall** be adjustable from 0.25G (approximately $[2.44 \text{ meters/sec}^2]^2$) to 5.0G, (approximately $[48.77 \text{ meters/sec}^2]^2$) in steps of 0.25G.

<u>Note:</u> *Although the value of Q is given in meters and seconds, it must be converted to the units of latitude, longitude, and altitude for use in the above extrapolation equations. The conversion factor for meters is given in §1.1.1.1.1.1.1.1.6, and meters and feet are related by 1 meter = 3.281 feet.*

1.1.1.1.1.1.1.1.10 Position Outlier Test

If the new reception is a 1090ES Position Message, then the following Outlier Test **shall** be performed. The Position Residual is defined by:

Position Residual = x(meas) – x(extr)

shall be compared with a position limit,

Position Limit = KP * SigmaP

where "KP" is an adjustable system parameter, and "SigmaP" is the joint positional sigma incorporating the measurement and filter positional sigma values. "SigmaP" is determined by:

$SigmaP = sqrt [Sig^2x(extr) + Sig^2x(meas)]$

"Sigx(extr)" is the extrapolated positional covariance matrix as defined in §1.1.1.1.1.1.1.1.9. "Sigx(meas)" is the covariance of the most recent position measurement as determined from its NAC_P value (see Table 3.2.3.2.1.1.2.3.6A), or its NUC_P value (see Table 3.2.3.2.1.1.2.3.6C).

The test is defined as:

Magnitude of Position Residual < Position Limit

If this test is passed for all three position components, then the Outlier Test is passed. For the two horizontal components, the values of "KP" **shall** be the same using the POSITION_OUTLIER_HORIZ_CONST configuration parameter, but for altitude, "KP" **shall** be a different parameter, POSITION_OUTLIER_ALT_CONST, that can have a different value. The nominal value of “KP” is nine (9), and its value **shall** be adjustable from 3 to 15 in steps of ONE (1).

If this Outlier Test is failed, this reception is not used except when required to be reported to the maintenance interface. In subsequent receptions, the value of DeltaT, calculated in §1.1.1.1.1.1.1.1.8, **shall** be based only on receptions that pass the Outlier Test.

1.1.1.1.1.1.1.1.11 Update When a Position Message is Received

a. If the new reception is a 1090ES Position Message, and if the Outlier Test has been passed, then the state **shall** be updated using the following formulas.

x(est) = x(extr) + alpha1 * (x(meas) - x(extr))

xd(est) = xd(extr) + beta1 * (x(meas) - x(extr))

where

x(meas) = the new position reception

alpha1 = Sig^2x(extr) / (Sig^2x(extr) + Sig^2x(meas))

beta1 = Sigxxd(extr) / (Sig^2x(extr) + Sig^2x(meas))

and where Sig^2x(meas) reflects the accuracy of the new position measurement. The value of Sig^2x(meas) is obtained from the most recent value of NAC_P or NUC_P as described in §1.1.1.1.1.1.1.1.4. The NAC_P or NUC_P value is converted to Sig^2x(meas) using the tables given in §1.1.1.1.1.1.1.1.6.

b. The covariance matrix **shall** be updated using the following formulas.

Sig^2x(est) = (1 - alpha1) * Sig^2x(extr)

Sig^2xd(est) = Sig^2xd(extr) - beta1 * Sigxxd(extr)

Sigxxd(est) = (1 - alpha1) * Sigxxd(extr)

1.1.1.1.1.1.1.1.12 Velocity Outlier Test

If the new reception is a 1090ES Velocity Message, then the Outlier Test **shall** be performed. The Velocity Residual is defined by:

Velocity Residual = xd(meas) – xd(extr)

shall be compared with a velocity limit,

Velocity Limit = KV * SigmaV

where “KV” is an adjustable system parameter, and “SigmaV” is the joint velocity sigma incorporating the measurement and filter velocity sigma values. “SigmaV” is determined by:

$$SigmaV = sqrt [Sig^2xd(extr) + Sig^2xd(meas)]$$

"Sigxd(extr)" is the extrapolated velocity covariance matrix as defined in §1.1.1.1.1.1.1.1.11. "Sigxd(meas)" is the covariance of the most recent velocity measurement as determined from its NAC_V value (see Table 3.2.3.2.1.1.2.3.6B), or its NUC_R value (see Table 3.2.3.2.1.1.2.3.6D).

The test is:

Magnitude of Velocity Residual < Velocity Limit

If this test is passed for all three components, then the Outlier Test is passed. For the two horizontal velocity components, the values of "KV" **shall** be the same using the VELOCITY_OUTLIER_HORIZ_CONST configuration parameter, but for vertical velocity, "KV" **shall** be a different parameter, POSITION_OUTLIER_VERT_CONST, that can have a different value. The nominal value of "KV" is nine (9), and its value **shall** be adjustable from 3 to 15 in steps of ONE (1).

If this Outlier Test is failed, this reception is not used except when required to be reported to the maintenance interface. In subsequent receptions, the value of DeltaT, calculated in §1.1.1.1.1.1.1.1.8, **shall** be based only on receptions that pass this Outlier Test.

1.1.1.1.1.1.1.1.13 Update When a Velocity Message is Received

a. If the new reception is a 1090ES Velocity Message, and if the Outlier Test has been passed, then the state **shall** be updated using the following formulas.

   x(est) = x(extr) + alpha2 * (xd(meas) - xd(extr))

   xd(est) = xd(extr) + beta2 * (xd(meas) - xd(extr))

   where

   xd(meas) = the new velocity reception

   $alpha2 = Sigxxd(extr) / (Sig^2xd(extr) + Sig^2xd(meas))$

   $beta2 = Sig^2xd(extr) / (Sig^2xd(extr) + Sig^2xd(meas))$

   and where $Sig^2xd(meas)$ reflects the accuracy of the new velocity information.

b. The value of $Sig^2xd(meas)$ **shall** be obtained from the most recent value of NAC_V or NUC_R as described in §1.1.1.1.1.1.1.1.4.

c. The NAC_V or NUC_R value **shall** be converted to $Sig^2xd(meas)$ using the tables given in §1.1.1.1.1.1.1.1.6.

d. The covariance matrix **shall** be updated using the following formulas.

$Sig^2x(est) = Sig^2x(extr) - alpha2 * Sigxxd(extr)$

$Sig^2xd(est) = (1 - beta2) * Sig^2xd(extr)$

Sigxxd(est) = (1 - beta2) * Sigxxd(extr)

Note: *After performing this update, the time registered position and velocity are available for reporting.*

1.1.1.1.1.1.1.1.14 Output Elements

a. The output elements are:

Position = x(est), y(est), z(est)

Velocity = xd(est), yd(est), zd(est)

Time = Time of Applicability accompanying this input

b. To obtain the output values of NAC_P and NAC_V from the covariance values, the following two tables **shall** be used. Where both horizontal and vertical covariance values are tabulated, the output **shall** be the highest value of NAC_P satisfying both horizontal and vertical conditions. Note that the horizontal limits are given in meters, so they must be expressed in the units of x and y (as described in §1.1.1.1.1.1.1.1.6 and §1.1.1.1.1.1.1.1.10).

Table 3.2.3.2.1.1.2.3.14A: Output NAC_P Values.

Sig(x) (Meters)	**Sig(z) (feet)**	**NAC_P**
< 1.2	< 7	11
< 4.1	< 25	10
< 12	< 75	9
< 38		8
< 76		7
< 230		6
< 380		5
< 760		4
< 1500		3
< 3000		2
< 7600		1
≥ 7600		0

Table 3.2.3.2.1.1.2.3.14B: Output NAC_V Values.

Sig(xd) (m/sec)	Sig(zd) (ft/sec)	NAC_V
< 0.12	< 0.8	4
< 0.41	< 2.5	3
< 1.2	< 8	2
< 4.1	< 25	1
> 4.1	≥ 25	0

1.1.1.1.1.1.1.1.15 Drop of Track

The track used to register position and velocity is maintained only as long as 1090ES ADS-B Messages are being received from that target. If 120 seconds passes during which there was no position or velocity reception, the track **shall** be dropped.

If a sequence of more than "KO" consecutive position outlier test (see §3.2.3.2.1.1.2.3.10) and velocity outlier test (see §3.2.3.2.1.1.2.3.12) failures has occurred on a given track, then the track **shall** be immediately dropped. "KO" is a configuration parameter defined by OUTLIER_FAILURES, whose nominal value is three (3), and its value **shall** be adjustable from 2 to 15 in steps of ONE (1).