RTCA Special Committee 186, Working Group 3
EUROCAE WG-51, SG-1

ADS-B 1090ES MOPS Maintenance

WG-3 Meeting #29
SG-1 Meeting #6
Joint Session

RTCA Headquarters
Washington DC
21 – 24 July 2009

Appendix E material
Air-to-Air Range as Limited by Power
In Response to Action Item 26-02

William Harman
MIT Lincoln Lab

Summary
The revised MOPS will contain new material on class A1 avionics having a single bottom antenna (A1S). WG-3 is planning to revise Appendix E to include class A1S. This working paper presents a performance analysis for A1S, and also a draft of a revised Appendix E.

The revised MOPS will contain new material on class A1 avionics having a single bottom antenna (A1S). This working paper presents a performance analysis for A1S, and also a draft of the revised Appendix E. The performance in Appendix E applies to interference-free conditions, where air-to-air range is limited by transmitter power and receiver sensitivity.

Performance Analysis. The first case considered here is A1S to A1, in which a transmitting aircraft is in class A1 and has a single bottom antenna, and a receiving aircraft is in class A1 equipped with top and bottom antennas. The analysis was done as a Monte Carlo simulation, including 1000 air-to-air encounters. Each run was done for a fixed value of air-to-air range. Random number generators were used for aircraft antenna gains, and transmitter power levels. After a run of 1000 encounters, the percentage of encounters having successful air-to-air surveillance is the result, which is compared with 95%. Then the value of air-to-air range was changed until achieving 95% successful performance. The result gives the power-limited range for this case.

Aircraft Antenna Gains. For this evaluation, aircraft antenna gains are characterized the TLAT antenna model (ref., TLAT report, Appendix D). Antenna gain G has a bell shaped distribution, generated as follows. Let x be a random variable, uniformly distributed between 0 and 1. Then

For x < 0.237	G(dB) = 2.76 + 0.5677 * 10 * log10(x)
For 0.327 <= x < 0.8	G(dB) = -4.8 + 25.42 * x - 39.354 * x^2 + 23.333 * x^3
Otherwise	G(dB) = 0.320 - 0.27813 * log10(1 - x)

The mean value of G is 0.50 dB, and the standard deviation is 2.70 dB. The bell shaped distribution of G is illustrated in Figure 1.

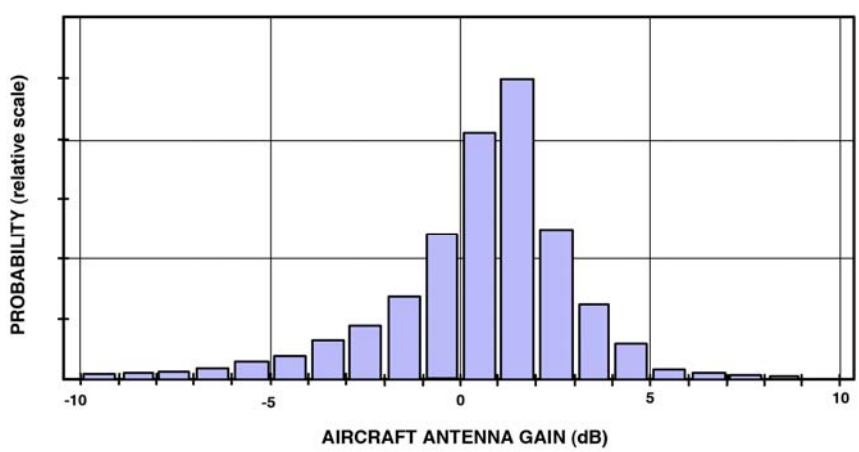

Figure 1. Probability Distribution of Aircraft Antenna Values.

Transmitter Power. For Class A1 transmitter power is required to be between 51 and 57 dBm referred to the transmitting antenna. That is a broad range (6 dB), and it seems reasonable to expect that manufacturers of A1S avionics will tend to favor the lower portion. For that reason, Appendix P in DO-260A modeled A1 transmitter power as uniformly distributed between 51 and 54 dBm. The same model is used in this analysis.

Receiver Sensitivity. Receiver sensitivity is characterized by Minimum Triggering Lever, or MTL, which is the power level at which reception probability is 90%. For Class A1, MTL is required to be -79 dBm or better referred to the receiving antenna. For this analysis, MTL was assigned to be exactly -79 dBm. Therefore the actual performance of typical A1S avionics will be somewhat better than the results calculated here.

Received Power. Received power is calculated as illustrated as follows (illustrated by an example).

Transmitter power (at antenna)	52.5	dBm
Transmitting antenna gain (nominal)	0	dB
Free space path loss (let R = 20 nmi)	-128.0	dB
Receiving antenna gain (nominal)	0	dB
Received power (at antenna)	-75.5	dBm

More generally, free space path loss is given by

Free space path loss = 20 log(4 pi R / wavelength)

In the simple example above, received power is stronger than MTL, so surveillance would be successful. Of course that is not true in every case because the antenna gains can be negative and the transmitter power can be lower than the mid-range value used in the table.

For each of encounter is the Monte Carlo simulation, received power is calculated as the sum of the items listed. There are two values of received power, corresponding to the top and bottom antennas. The greater of these is used to determine if the receptions are sufficiently strong.

Successful Air-to-Air Surveillance. When received power equals MTL, reception probability is 90%, and surveillance is successful. Given the ADS-B transmission rates, 2 position messages per second and 2 velocity messages per second, it follows that the reception probability can be substantially lower than 90% for successful air-to-air surveillance. The determination of the needed value of reception probability is the subject of Appendix K in DO-260A and also Appendix B in the report "1090 MHz Extended Squitter Assessment Report", by the FAA and EUROCONTROL Experimental Center, June 2002. The results show that reception probability of 0.07 is sufficient to provide air-to-air surveillance equivalent to the ADS-B MASPS (DO-242A) standards for Nominal Update Interval of 12 seconds. This is the basis used in this Monte Carlo analysis for each of the 1000 encounters.

For convenience, a copy of Appendix B in included here at the end this working paper.

The following figure shows typical behavior of an ADS-B receiver in the vicinity of MTL. This data was obtained from bench tests at the FAA Tech Center in January 2009. Between the MTL point (90%) and the point of successful air-to-air surveillance (7%), the difference is 4 dB. For each of the N encounters in the Monte Carlo simulation, a determination is made of whether the received power is stronger than MTL - 4 dB, and if so the surveillance is considered successful.

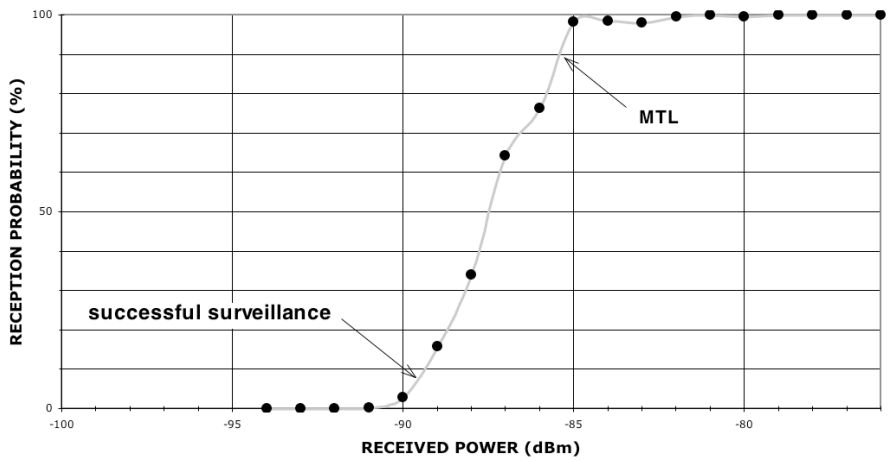

Figure 2. Reception probability for weak signals.

Final range calculation. Beginning with a given arbitrary value of air-to-air range, and having determined whether each of the simulated encounters is successful, the percentage of all encounters which are successful is determined. That percentage is compared with 95%. Then the simulation is run again with a different value of range, and that process is repeated until the result is 95%. That result is the calculated air-to-air range for this case. The result is:

Air-to-air range = 47 nmi for class A1S to class A1

The same process was used for class A2 receivers and class A3 receivers. The MTL models for those two classes were the same as in Appendix P of DO-260A. Specifically,

For class A2, the MTL model is uniform between 51 and 54 dBm (at antenna).
For class A3, the MTL model is uniform between 53 and 56 dBm (at antenna).

Those results are:

Air-to-air range = 47 nmi for class A1S to class A2
Air-to-air range = 75 nmi for class A1S to class A3

Updating the text and values in Appendix E. Although I was one of those who wrote Appendix E many years ago, looking at it now, I see that the material is poorly presented. It has

two tables, but curiously the second table appears in italics in a note. A more normal presentation would simply show two tables. Furthermore, the two tables both give air-to-air range values but the values are substantially different. The difference is caused by the fact that the calculations for the first table are oversimplified. After WG3 has finished discussing these issues in detail, I think it would be better to present a single table to our readers -- omit the inaccurate table, and simply show the results that we believe are accurate.

I also noticed a blatant error in Appendix E in DO-260A. Whereas classes A1 and A2 have the same transmitter power requirements and the same receiver MTL requirements, the air-to-air range values are quite different, which is incorrect. The error may have originated from a last-minute change to the MTL requirements without remembering to change the range values in Appendix E.

I have drafted an improved version of Appendix E, below. I also re-calculated the air-to-air ranges for the cases shown in DO-260A using the Monte Carlo method documented in this paper.

= = = = =

Appendix E. Air-to-Air Range as Limited by Power

The requirements for transmitter power and receiver sensitivity are different for the different avionics classes. These requirements, given in section 2.2, can be summarized as follows.

Table E-1. Summary of Transmitter and Receiver Requirements

Avionics Class	Transmitter Power dBm at antenna	MTL dBm at antenna
A0	48.5 to 57	-72 or lower
A1	51 to 57	-79 or lower
A2	51 to 57	-79 or lower
A3	53 to 57	MTL(90%) = -84 or lower MTL(15%) = -87 or lower

As a result of these differences, the maximum air-to-air range differs from class to class. Following are the values of air-to-air range as limited by these power level requirements. In other words, these are the air-to-air ranges in an interference-free environment. These results apply to a receiving aircraft having the worst-case value of MTL for that class, and they apply to 95 percent of transmitting aircraft whose transmitter power levels are characterized by the models described in Appendix P. Aircraft antenna gains for both transmitting aircraft and receiving aircraft are characterized by the TLAT antenna-gain model (ref., "Technical Link Assessment Report", Appendix D, RTCA Free Flight Select Committee, Eurocontrol ADS Programme, March 2001). For Class A0 and Class A1S, the results apply to a single bottom mounted antenna. For the other classes, the results apply to top-bottom antenna diversity.

Table E-2. Air-to-Air Range as Limited by Power

Avionics Classes	Air-to-Air Range NM
A0 to A0	10
A1S to A1	47
A1S to A2	47
A1S to A3	75
A1 to A1	66
A2 to A2	66
A3 to A3	140

= = = = =

The following is an excerpt from the report, "1090 MHz Extended Squitter Assessment Report", FAA and EUROCONTROL Experimental Center, June 2002.

Appendix B
Criteria for State Vector Update Rate

Purpose

An initial evaluation of Extended Squitter performance in Europe used a criterion in which it was assumed that surveillance update rate would be unsatisfactory unless the probability of receiving both position and velocity in a 12 second period is at least 95 percent. When it was found that the system performance calculated in this initial analysis was very poor, attention was then given to several conservative assumptions that were used in the analysis.

Appendix K of the Extended Squitter MOPS (DO-160) focuses on the update-rate characteristics of Extended Squitter and compares this system to the baseline performance defined in the ADS-B MASPS (DO-242). In Extended Squitter, receptions times are random rather than being periodic, and position and velocity are received separately. It is shown in the appendix that these difference in timing characteristics should be considered when evaluating the surveillance update-rate performance. More specifically, it is shown that a requirement for receiving both a position and a velocity in a nominal surveillance period would yield significantly better update performance than the baseline system. Therefore a requirement for receiving both position and velocity would not be appropriate for system evaluation.

The analysis in Appendix K applies to short ranges for which the nominal update period is 3 seconds. The purpose of this appendix is to extend the analysis to the long range regime in which the nominal update period is 12 seconds.

Formulation

This study is formulated as an encounter between two aircraft, in which air-to-air surveillance is occurring by means of Extended Squitter transmissions from one aircraft, being received by the other aircraft. The scenario represents enroute flight, and the encounter begins when the air-to-air range is 40 nmi, for which the nominal update period is 12 seconds. One aircraft makes a 20 degree turn as illustrated in Figure 1. The projected horizontal separation becomes less than 5 nmi after the turn, whereas the aircraft would have been safely separated if the aircraft had not turned. The speeds of both aircraft are 500 knots, and the turn rate is one degree per second. As the turn occurs, surveillance information is received at the other aircraft, and eventually the received information indicates that a conflict is occurring. Limitations in surveillance update rate have the effect of delaying the detection of the conflict.

To assess performance both of Extended Squitter and the baseline ADS-B system, a Monte Carlo simulation was used. To represent Extended Squitter receptions, each transmission is characterized by a constant reception probability. Therefore the delay in receiving information about the change in separation is randomized according to the actual reception times, which are random. To represent the baseline ADS-B system, the phasing of the 12 second nominal update periods is random relative to the beginning and ending of the turn.

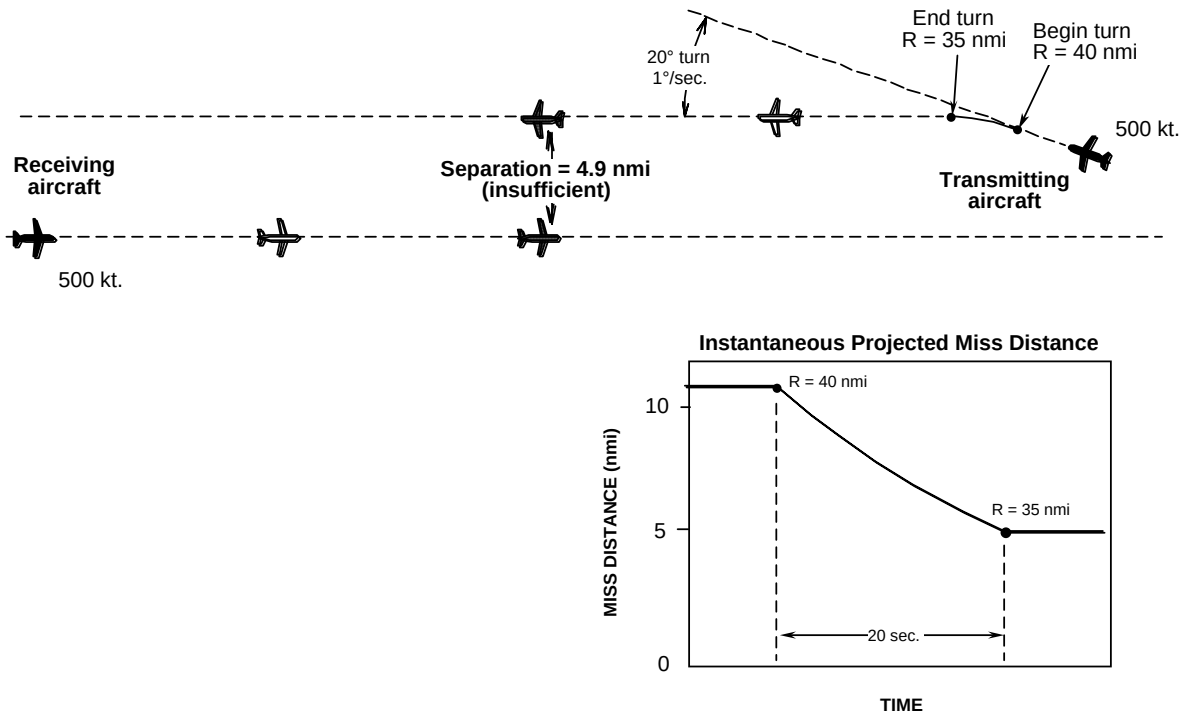

Figure 1. Scenario for Assessing Update Rates

In assessing performance, the simulation of the avionics on the receiving aircraft uses the received information to estimate the projected horizontal separation, which is called the miss distance. Figure 1 shows that true value of the instantaneous projected miss distance, against which the simulation results can be compared.

Results

Simulation results for a typical encounter are shown in Figure 2. Because of the random conditions, the behavior will change from run to run, but some significant differences can be seen in just one run, as shown here. Note that during a 12 second period, the projected miss distance changes considerably. Therefore for the baseline system, the 12 second spacing of updates will typically cause a significant delay in the time when the receiving aircraft becomes aware of the change in separation. The amount of this delay depends on the timing of the ADS-B transmissions relative to the turn timing. In this example (the upper plot) the delay is about 5 seconds. This delay can be more or less depending on the timing of the surveillance.

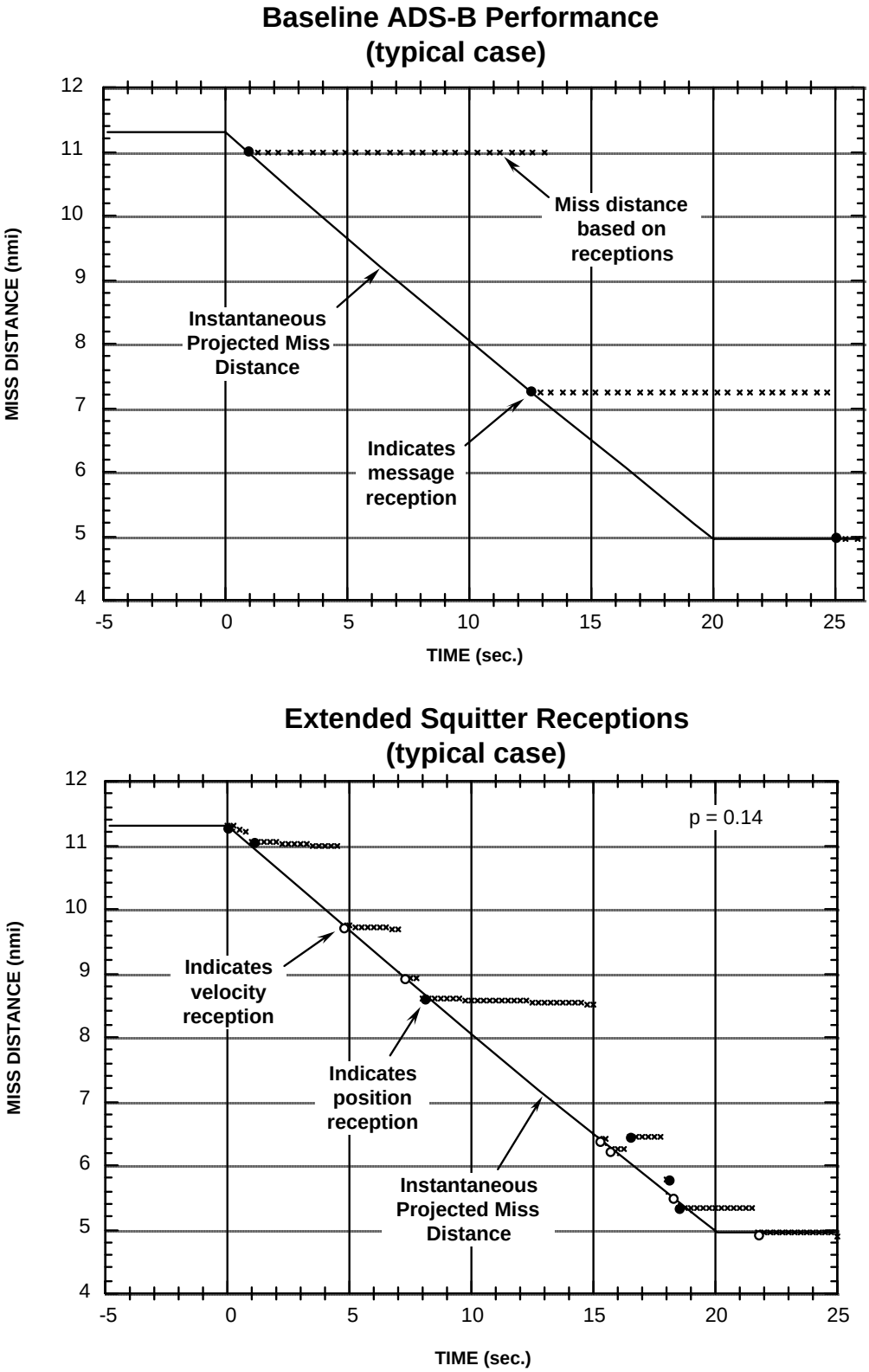

Figure 2. Simulation Results in Typical Cases.

The random timing of Extended Squitter is seen, in the lower plot, to provide typically much more frequent information about the change in miss distance. For this run, the per squitter reception probability was set equal to 0.14, which is the value that would provide 0.95 probability of receiving both position and velocity in 12 seconds. That is,

$$P = [1 - (1 - p)^{24}]^2$$

For $P = 0.95$, the solution is $p = 0.14$

Looking closely, one can see that every velocity reception provides nearly as much information as if it had included position along with velocity. One can also appreciate in this scenario that position inaccuracies and delays have a very small effect relative to velocity inaccuracies and delays.

Statistical results from multiple runs have been generated and are presented below, but even the Figure 2 results from a single run provide the answer to the main question. Seeing that a single velocity reception is essentially as informative as a position and velocity together, it follows that a criterion requiring both position and velocity reception in 12 seconds would be overly conservative.

The following point of view may be helpful in providing insight along with the results in Figure 2. Consider a particular 12 second period, and consider the moderately unlikely event that no position information was received during this time. According to the conservative criterion, any receptions during this time are of no value, but in reality some velocity information may have been received. In fact the average number of velocity receptions during this 12 seconds is

$$\text{Average number} = 24 * (0.14) = 3.4 \text{ velocity receptions}$$

Note that the randomness of velocity reception and position reception are independent, so that even during this unlucky period, it is nevertheless likely that multiple velocity receptions occur. Therefore, even during this low-probability event, surveillance is typically updated accurately.

Running the simulation multiple times makes it possible to observe the behavior including rare events. Figure 3 shows the 95-percentile miss distance error together with typical single runs for comparison.

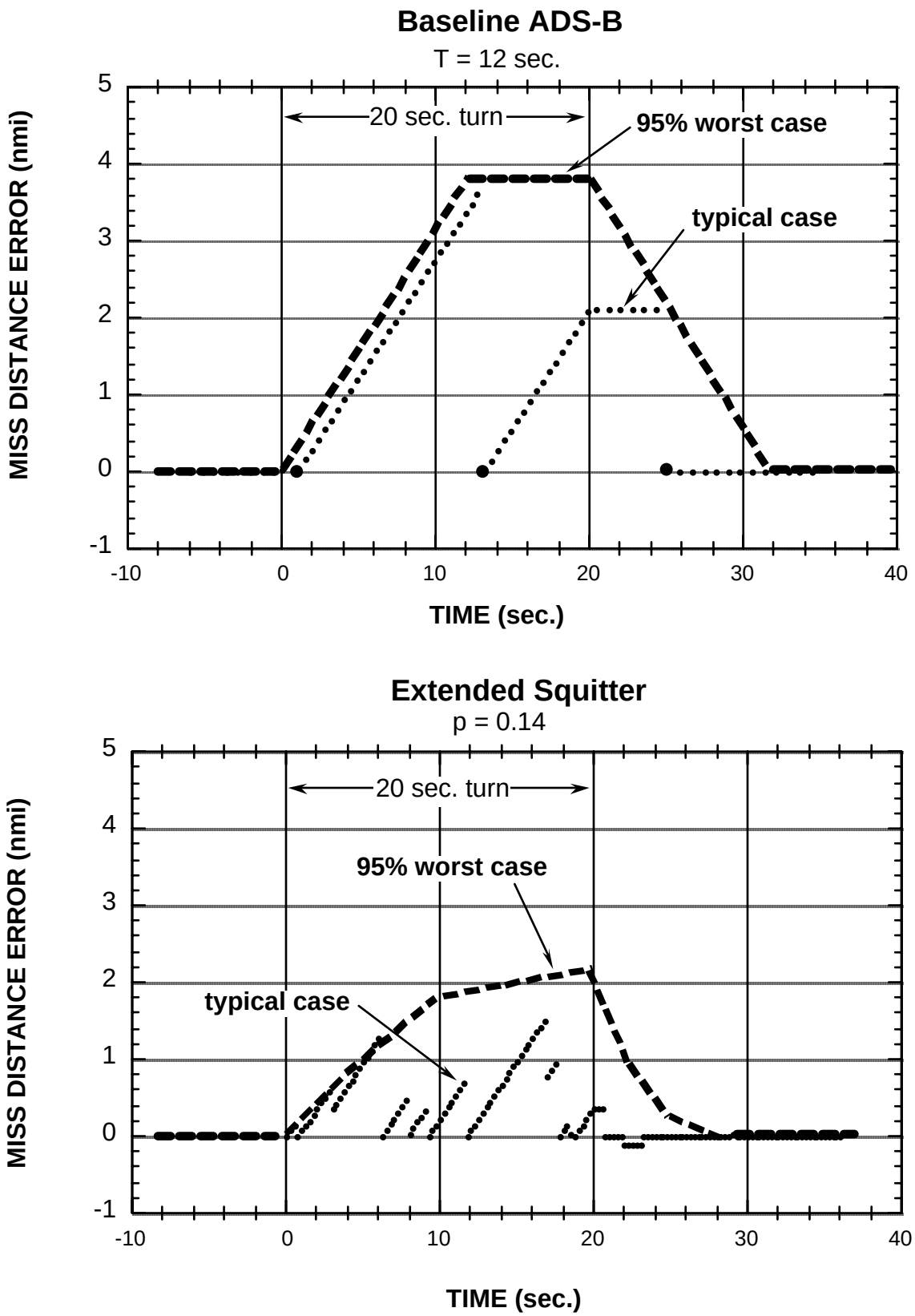

Figure 3. Results for 95-Percentile Worst Case

These results show explicitly that the surveillance update performance is made significantly better by the random timing characteristics of Extended Squitter. This is the same principle identified in MOPS Appendix K, except shown here for long range surveillance where the nominal update period is 12 seconds.

Two Turns

It is also interesting to consider a scenario in which the transmitting aircraft makes two turns instead of one. This scenario introduces the possibility that the receiving aircraft may detect the end of the first turn, and be less able to detect the second turn when the horizontal separation actually becomes insufficient. A specific two turn scenario is considered, for which the results are illustrated in Figure 4. The transmitting aircraft makes two 20 degree turns, separated by a 10 second straight section.

The results are similar to the behavior seen above. Depending on the phasing between the 12-second periodic reports in the baseline system, an error develops simply because time has passed while the geometry is changing. Given that the error can vary depending on phasing, there is a substantial uncertainty in the actual projected miss distance.

Specifically, the results show that by the end of the 10 second straight section, the error has become small, but that has little effect on the ability to detect the worsening separation. As soon as the second turn begins, the behavior is essentially the same as in the simpler case described above. In summary, given the ADS-B performance of the baseline system in this long-range, 12-second regime, received information indicates that some turning has happened but does not indicate more specifically that a turn has begun and ended. This update rate standard in the MASPS is based on the long-range conditions and the fact that there is a substantial amount of time in which the situation can be resolved.

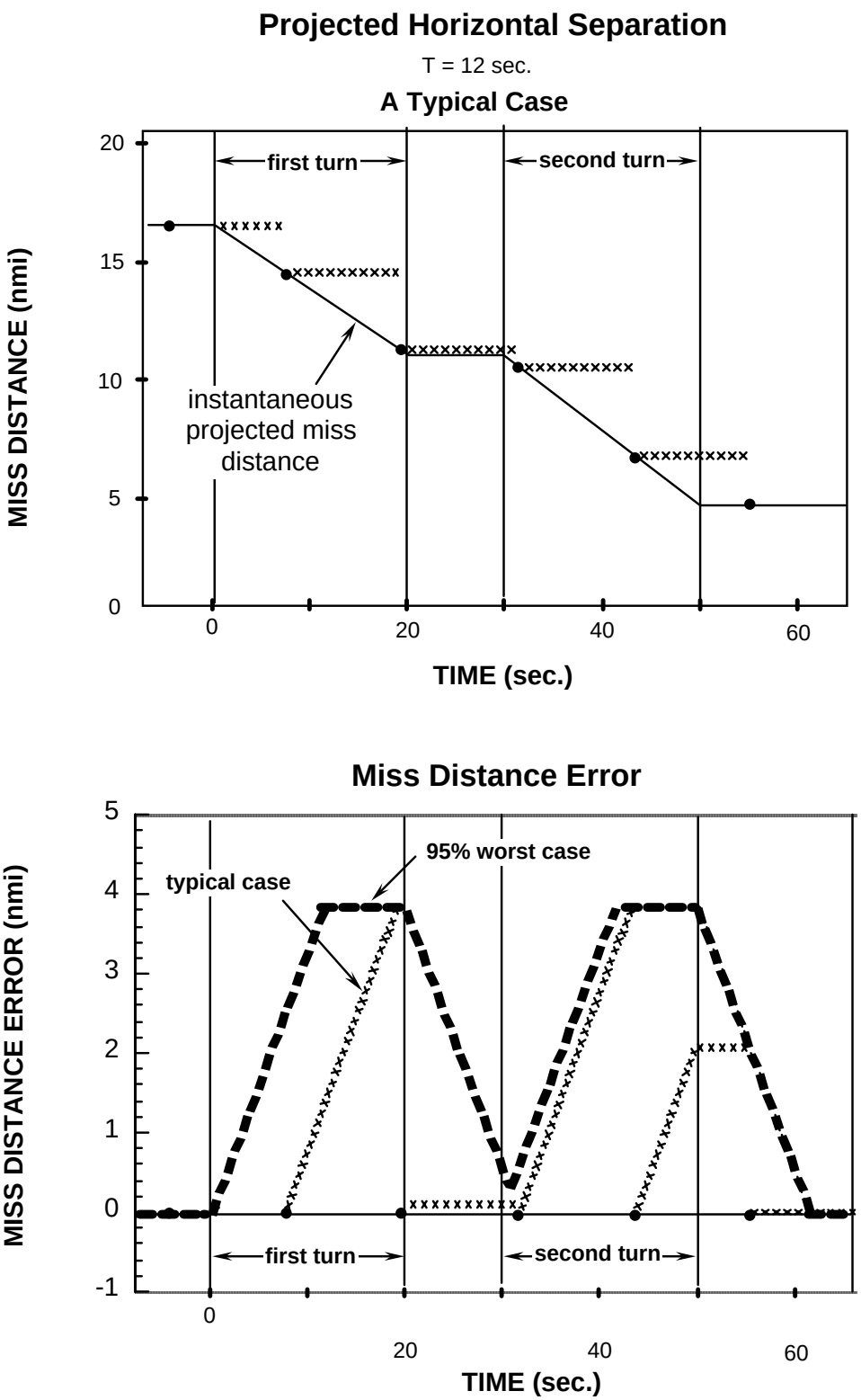

Figure 4. Behavior For Two Small Turns

Seeing that a position-and-velocity criterion would be inappropriate, similar calculations have been made for lower values of reception probability, p. Two other values were considered:

Position-and-velocity in 12 sec. => p = 0.14
Velocity in 12 sec. => p = 0.117
Position-or-velocity in 12 sec. => p = 0.061

The third criteria considered here is a natural choice because the report generation function of and Extended Squitter receiver generates a state vector report upon the reception of each reception, whether position or velocity. The results are summarized in Figure 5, which shows the average error in projected miss distance, averaged over 32 seconds, consisting of the 20 second turn and an additional 12 seconds The results indicate that the intermediate value, p = 0.117 is also inappropriate in the sense that performance is much better than the baseline. The position-or-velocity criterion, p = 0.061, yields Extended Squitter performance nearly the same as the baseline.

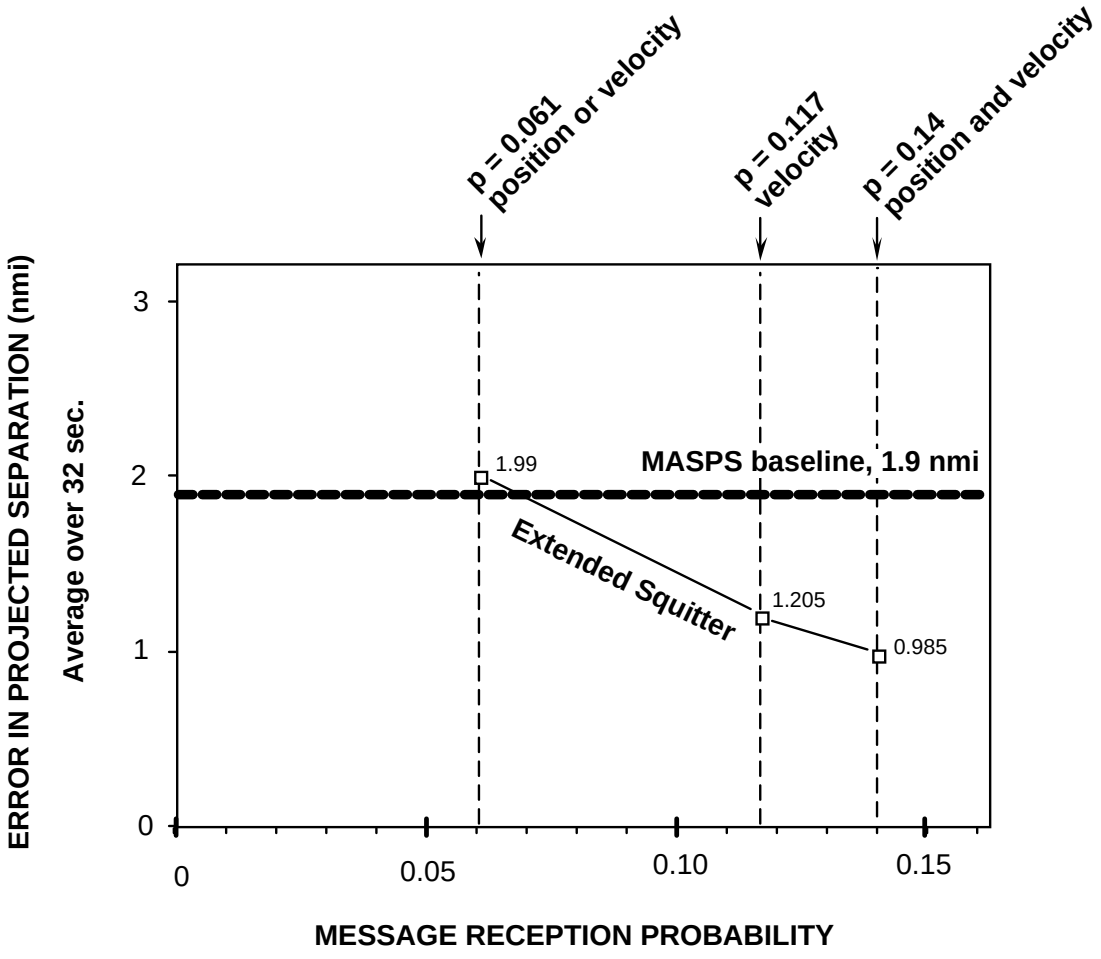

Figure B-5. The Effect of Reception Probability.

Conclusion

The simulation and analysis of long-range surveillance for which the nominal surveillance update period is 12 seconds shows the same principle as was previously found for short range surveillance. In evaluating Extended Squitter, a criterion requiring both position and velocity to be received in 12 seconds would not be appropriate because of the random timing characteristics of Extended Squitter, which have a beneficial effect on the updating of surveillance information.