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16 – 19 June 2009

RTCA Special Committee 186, Working Group 3
EUROCAE WG-51, SG-1

ADS-B 1090ES MOPS Maintenance

WG-3 Meeting #28
SG-1 Meeting #5
Joint Session

EUROCAE Headquarters
Malakoff, near Paris France
16 – 10 June 2009

Report on Action Item 27-10 for DO-260B/ED-102A: SIL Redefinition

George Ligler, PMEI
Supporting the FAA SBS Program Office

Summary
This Working Paper addresses a summary of two teleconferences held on the subject of SIL Redefinition as a result of Action Item 27-10.

Since our meeting in Chicago, two teleconferences were held to continue to develop consensus on the redefinition of SIL. These teleconferences built upon the agreements reached in Chicago and resulted in a consensus recommendation on remaining major aspects of the subject.

Attached are the following 5 components:

1. The Background Memorandum for the June 2 call.
2. A memorandum summarizing the results of the June 2 call and discussing the actions for the June 12 teleconference.

3-5.Three papers actioned in the June 2 call and discussed during the June 12 teleconference.

Participants in the June 12 call included Joel Wichgers, Don Walker, Jorg Steinleitner, Larry Kenney, Stan Jones, Rich Jennings, Chip Bulger, Tom Pagano, Jim Davis, Michael Garcia, Kirk Schueler, Dean Miller, and George Ligler. Persons who participated in the June 2 call but were unable to be part of the June 12 teleconference included Bruce DeCleene, Bob Saffell, and Kevin Vanderwerf.

The consensus of the June 12 call re redefinition is SIL is as follows:

1. Design assurance/avionics integrity (e.g., MAJOR, MINOR) is to be treated in a manner independent from the specification of the Probability of the NIC containment radius.

2. Design assurance/avionics integrity is to cover the entire ADS-B OUT equipment chain with regard to the reported position (i.e., position source, ADS-B avionics, any intermediate data concentrators).

3. The Probability for the NIC Containment Radius is to continue to be encodable as one of 10^{-7}, 10^{-5}, 10^{-3}, or unknown on either a per hour or per sample basis. Per hour or per sample is to be expressly indicated.

4. The MOPS should have a NOTE recommending that both the NIC and the Probability of the NIC Containment Radius should be set to "unknown" if the ADS-B position source does not supply an output certified to provide an indication of the integrity of the reported position (e.g., such as a GNSS HPL).

The teleconference attendees did not agree to a specific encoding for one or more 1090ES ADS-B message fields to implement this consensus. Some participants indicated a preference for having separate Design Assurance and Probability of NIC Containment Radius (also called "the new SIL") fields. It was also pointed out that the consensus could be implemented within one message field.

I would like to thank all participants for their work in achieving this consensus recommendation to WG-3/SG-1.

ATTACHMENT A

RTCA Special Committee 186 Working Group 3/EUROCAE WG51 Subgroup 1
Background for June 2, 2009 Teleconference on NIC Containment
Radius Considerations

In the Working Group 3/Subgroup 1 joint meeting held in Chicago May 13-15, 2009, a proposal to redefine SIL and, concomitantly, the manner in which the containment radius for NIC is specified was discussed by participants. The following is an excerpt from the Meeting Summary on these discussions.

5.7 The Meeting continued with the review of Working Paper 1090-WP27-18 under Agenda Item #5m by Chip Bulger and Richard Jennings for the discussion of the FAA AIR-130 and SBS Program Office proposal to redefine the SIL and NIC parameters. After review of the entire Working Paper, it came down to four basic recommendations. Recommendation #1 was to "*Move the containment radius probability from SIL to NIC (agreed) and fix the containment radius probability at 10^{-7} (not agreed).*" Recommendation #2 (agreed) was to "*Include the entire avionics chain in defining the design assurance/avionics integrity parameter (SIL).*" Recommendation #3 (agreed) was to "*Redefine SIL to Support Failure condition.*" Finally, recommendation #4 (initially agreed but later questioned, see below) was to "*Add a GNSS/non-GNSS Bit.*" It was agreed that the containment radius for the ADS-B position source should be expressed in a different parameter than avionics integrity. Consistent with this agreement, there was further consensus that we have a design assurance parameter that covers the entire ADS-B OUT equipment chain. The MS WORD part of the Working Paper was revised during the meeting by George Ligler to 1090-WP27-18R1 to reflect an intermediate point in the above discussions. The revised Working Paper was further discussed later in the Meeting in an effort to come to agreement on all SIL-related issues. It was agreed that ADS-B position source containment radius probabilities would continue to be encoded as one of 10^{-7}, 10^{-5}, or 10^{-3}. Nonetheless, two items remained without consensus: (1) whether the containment radius probability should be encoded as per hour or alternatively as per hour or per sample; and (2) whether there should be a bit transmitted to indicate whether the containment radius was per hour or per sample. As a result of this discussion, George Ligler accepted **Action Item 27-10** to lead a teleconference that was scheduled for 9:00am EDT on 2 June 2009 in an effort to resolve the "per hour or per sample" and related "per/hour or per/sample bit" issues. George indicated that he would put out a paper framing the 2 June discussion in advance of the teleconference.

Reviewing the bidding, we have agreed to the following:

(1) Design assurance/avionics integrity (e.g., MINOR, MAJOR) is to be treated in a manner independent from the specification of the NIC containment radius.

(2) Design assurance/avionics integrity is to cover the entire ADS-B OUT equipment chain (i.e., position source, ADS-B avionics, any intermediate data concentrators).

(3) The Probability for the NIC Containment Radius is to continue to be encodable as one of 10^{-7}, 10^{-5}, or 10^{-3}.

Specific bit-wise encodings for the NIC Containment Radius and Design Assurance/Avionics Integrity parameters have not yet been agreed, although several alternatives are on the table.

Review of the above meeting summary as well as of Paper 1090-WP27-18R1 will show that consensus has not yet been reached on two points:

(4) Whether the Probability for the NIC Containment Radius is to be specified on only a per-hour basis, or whether that probability is encodable as being either a per-hour or per-sample quantity.

(5) If the decision is made that we can specify the Probability for the NIC Containment Radius as either per-hour or per-sample, whether or not a bit should be included in the ADS-B broadcast to indicate which of these alternatives is appropriate for the ADS-B position source currently being used.

The June 2, 2009 teleconference (9AM-12PM Eastern time: teleconference details to follow) has as its sole objective the resolution of the two points above.

The motivation for being able to specify Probabilities for the NIC Containment Radius on a per-sample basis is to be able to use certain non-GNSS position sources for ADS-B. The rationale given in Paper 1090-WP27-18R1 for the per-sample encoding as well as for the "per-hour/per-sample bit" is as follows:

> **"Issue 3) Addressing position sources other than GNSS**
>
> Issue: The standards must accommodate position sources other than GNSS, and the detailed requirements need to be defined[1]. Inertial coasting and DME/DME positioning have different failure characteristics and a containment radius per hour is not available.
>
> Discussion: The current NIC and SIL definitions address non-GNSS position sources by providing a containment radius with a given per sample probability, rather than per hour. RNP-certified FMSs provide an ANP or EPE associated with these position sources, and can be used to derive a NIC to broadcast. In all cases, the NIC derived from the ANP/EPE is determined by extrapolating based

[1] The separation analysis for positioning sources other than GNSS is not complete. The RFG analysis included assumptions for the failure effects of GNSS. GPS with receiver autonomous integrity monitoring (RAIM) was considered to be the limiting case, and was specifically addressed in the analysis.

on an assumed fault-free distribution. As such, the extrapolated value can be selected to be a 10^{-5} or 10^{-7} value. For example, the Boeing 777 RNP Capabilities document explains that the 95% radial accuracy is 0.86 of the output ANP. Extrapolating on a Rayleigh distribution, the aircraft could broadcast a 10^{-5} value or a 10^{-7} without impacting the system itself. Extrapolating to 10^{-7} would provide some consistency with the GNSS position source requirement, so that the only difference would be per hour or per sample.

As a related issue, there may be future applications where the distinction between the GNSS per hour requirement and non-GNSS per sample requirement becomes important. A new positioning source indication would be useful to provide a means for the ADS-B In equipment to distinguish the position source used by the other aircraft. As an example, it is expected that the potential for common-mode failures caused by satellites could affect the NIC requirements for delegated separation, but that common-mode failure would not exist between two aircraft using different position sources."

During our teleconference on June 2, this rationale will be discussed in further detail. For example, we will discuss the use of DME-DME as a specific example of a candidate ADS-B position source that, with signals that are monitored on the ground by the ANSP, could provide a per-sample NIC Containment Radius without complex calculations in the aircraft involving, e.g., the current DME ground station geometry experienced by the aircraft or the failure rate of DME ground stations. Additionally, we will discuss recent confirmation from Dr. Mahesh Jeerage of Honeywell that the containment radius from an RNP-certified Flight Management System is an instantaneous (as opposed to per-hour) value that cannot readily be extrapolated to a per-hour value.

As stated in the Chicago meeting, the intent of these aspects of Paper 1090-WP27-18R1 is to provide a framework within DO-260B/ED-102A that supports the certification and appropriate use of non-GNSS ADS-B position sources as we fill the "analytical gap" (with regard to non-GNSS ADS-B position sources) referred to in the above footnote within the paper.

ATTACHMENT B

From: George Ligler <ligler1@earthlink.net>
To: gary.ctr.furr@faa.gov
Subject: Fw: 1090 ES MOPS Discussion on SIL Containment Radius-12 June
Date: Jun 4, 2009 9:08 PM

Gary:

Thanks in advance for forwarding the e-mail I've just sent to the 1090 MOPS list to the SG1 list.

Best Regards,
George

-----Forwarded Message-----
>From: George Ligler <ligler1@earthlink.net>
>Sent: Jun 4, 2009 9:06 PM
>To: 1090mops@its.tc.faa.gov, bruce.decleene@faa.gov, jmwichge@rockwellcollins.com
>Subject: 1090 ES MOPS Discussion on SIL Containment Radius-12 June
>
>Dear Colleagues:
>
>As we discussed at our teleconference on 2 June, we will hold a follow-up discussion on Friday, June 12 from 9 a.m. to 11 a.m. Eastern time. By way of review, we agreed during our 2 June discussion that the Probability for the NIC Containment Radius needs to be encodable as either a "per-hour" quantity (e.g., for GNSS as an ADS-B position source) or as a "per-sample" quantity (e.g., for an RNP-certified FMS as an ADS-B position source). What we agreed needs to be discussed further is the manner in which the per-hour/per-sample aspect of the Probability for the NIC Containment Radius will be encoded. Three alternatives are on the table, and brief white papers on each alternative will be prepared for our June 12 discussion. The three alternatives are as follows:
>
>1. Use one bit to indicate per-hour/per-sample. Chip Bulger has agreed to prepare the white paper on this alternative, and in doing so will provide a concise definition of what "per-sample" means in the context of the Probability for the NIC Containment Radius.
>
>2. Employ an ADS-B message field, particular values of which will denote particular ADS-B position sources, and some values of which will be reserved for the future. Stan Jones, George Ligler, and Bob Saffell have agreed to prepare the white paper on this alternative.
>
>3. Use one bit to indicate per-hour/per-sample, and additionally employ an ADS-B message field to indicate the correlation time associated with an ADS-B position source that has a Probability for the NIC Containment Radius that is a per-sample quantity. Don Walker and Kevin Wanderwerf have agreed to prepare the white paper on this alternative.
>
>For those of you who have not had the opportunity to review the Background paper that framed our June 2 discussion, a copy of that

paper is attached. We agreed that the three white papers would be provided to either myself or Gary Furr by close of business on Thursday, June 11 for distribution that evening to teleconference participants. The results of our June 12 teleconference will be presented to the WG-3/SG-1 meeting to be held in Paris on June 16-19.
>
>The teleconference coordinates are as follows:
>
>Number to call: 888-481-3032
>Participant Code: 11959028.
>
>I look forward to our discussion.
>
>Best Regards,
>George Ligler

Attachment C

1090-WP28-xx
12 June 2009

RTCA Special Committee 186, Working Group 3
EUROCAE WG-51, SG-1

ADS-B 1090ES MOPS Maintenance

WG-3 Meeting #28
SG-1 Meeting #
Joint Session

Paris, France
16 June 2009

Per Hour / Per Sample Bit

Bruce DeCleene and Chip Bulger, FAA AIR-130

Summary
This Working Paper addresses the addition of a per hour/per sample bit that will indicate whether the position source in use bases the probability of exceeding the NIC containment radius on a per sample basis or a per hour basis.

1. Introduction:

DO-260A allowed SIL to indicate the per hour probability or the per sample probability of the position exceeding the NIC containment region. SIL is being revised in DO-260B, and WG#3 is proposing that the per hour and per sample probabilities should still be allowable. However there is a suggestion that there be an ability to transmit whether the probability of exceeding the NIC containment region is based on a per hour or per sample basis, considering that the method by which these values will be used could be different for some applications.

For position sources using GNSS signals (including integration with inertial reference units - IRUs), the NIC is determined from the HPL which relates to a probability of exceeding an integrity containment radius of 10^{-7} per hour.

For position sources using LOC and DME signals, and IRU position sources (depending on previous updates) the probability of exceeding the integrity containment radius is currently available on a per sample basis.

Although GNSS is expected to be the most common position source for ADS-B, there is potential value in providing methods to use other position sources.

2. Discussion

This paper proposes adding a one bit value that indicates whether the position source calculates its probability of exceeding the NIC containment radius on either a per hour or per sample rate.

2.1 Operational Need

Why is this bit necessary? ATC separation services (or future applications) could need to know whether the probability of exceeding the integrity containment radius is based on per hour or per sample. Currently, all analysis by the Requirements Focus Group (RFG) for the RAD document and all analysis completed by the FAA's Separation Standards Working Group (SSWG) has been based on the GNSS per hour probability. The INS or DME based RNAV per sample probability is still being evaluated.

2.2 Proposed Definitions

• Per Hour: The probability of a reported geometric position laying outside the NIC containment radius is defined as the probability of a single position being outside the NIC containment radius in any given hour.

• Per Sample: The probability of each reported geometric position laying outside the NIC containment radius.

2.3 Determining Per Hour or Per Sample for Position Sources

The primary motivation behind this proposal is to accommodate existing RNAV implementations and retain a performance-based implementation of ADS-B transmissions. Currently, GNSS position sources make available an HPL based on 10-7 per hour, while ground-based RNAV position sources with RNP capability make available an estimate of position uncertainty (ANP or EPE). Accommodating these implementations is important in ensuring that ADS-B Out can be implemented for a variety of position sources.

To accommodate future position sources, consideration must be given to how to determine which definition to apply to each position source. The following requirement is proposed:

The selection of a NIC as a per hour or a per sample probability must be determined as part of installation and integration of the system. It must be determined for each position source used in the ADS-B broadcast. Both fault-free and faulted signal-in-space system performance must be considered.

A) Fault-free system performance: The fault-free system performance should be considered. The fault-free performance should be considered by characterizing the distribution of errors to a probability commensurate with the SIL_{NIC} (term to be determined). The radial errors can typically be characterized by a Rayleigh distribution, although the actual error distribution is elliptical (not circular), and the fault-free NIC can be based on extrapolating from accuracy metrics.

B) Faulted signal-in-space performance: The effects of potential, latent failures in the signal-in-space should be considered.

- The analysis should consider any potential latent conditions that are more likely than SIL_{NIC}.on a per hour basis. In other words, for a SIL_{NIC} of 10^{-7} per hour, potential latent faults more likely than 10^{-7} per hour need to be considered.
- For each potential failure condition, the monitor characteristics should be considered. Some systems have monitors that are very effective at truncating the tails of error distributions, such as the DME signal monitors which are very effective at detecting out-of-tolerance reply delays. Other systems, such as GPS, have failure conditions without any real-time monitor.
- Taken together, the NIC-under-failure is based on the aggregate of the likelihood of failure and the monitor characteristics. This parameter should not consider failures of aircraft systems themselves, as they are addressed in the SIL_{system} (term to be determined).

For each position source, the NIC should be based upon the larger of the fault-free NIC and the NIC-under-failure. Based on whichever is larger, the per sample (if fault-free) or per hour (if faulted) parameter should be set.

The following table provides guidance for typical position sources that can be used in lieu of detailed analysis. The larger term is highlighted.

Table – Typical Position Source Considerations

Position Source	**Fault-Free NIC**	**NIC-Under-Failure**	**SIL_{NIC}**
GNSS (including when augmented by IRU, LOC, other)	Extrapolation of HFOM	Equivalent to HIL/HPL	Per Hour
DME/DME	Extrapolation of EPE (e.g., ANP, RNP)	Constrained by signal monitoring.	Per Sample
DME/DME/IRU	Extrapolation of EPE (e.g., ANP, RNP)	Constrained by signal monitoring.	Per Sample
DME/DME/LOC	Extrapolation of EPE (e.g., ANP, RNP)	Constrained by signal monitoring.	Per Sample
VOR/DME	Not available (not able to validate assumptions on VOR signal accuracy except on pre-published VOR routes)	Not available (out-of-tolerance performance permitted off of pre-published VOR routes)	Not available
IRU only	Extrapolation of EPE (e.g., ANP, RNP)	Not applicable*	Per Sample

* For IRU only, there is no signal-in-space fault condition. However, typically IRU-only positioning retains some dependency on prior position updating from external sources, and the characteristics of that mode should be considered when determining NIC when coasting only on IRU.

3. Proposed Changes to DO-260A

<u>A single-bit proposal is the most efficient way to indicate this, as it is a two-state variable. That bit would be defined as:</u>

Bit	*Definition*
0	*Probability of exceeding NIC containment radius is based on per hour*

1	*Probability of exceeding NIC containment radius is based on per sample*

c. All Options: Add requirements to Section 2.2

d. All Options: Add requirements to Section 2.4 (Verification)

e. All Options: Modify Appendix A as appropriate

ATTACHMENT D

RTCA SC-186 WG-3/EUROCAE WG51 SG1

White Paper for SIL/NIC Considerations

Identification of ADS-B Position Source

June 11, 2009

Bob Saffell, Stan Jones, and George Ligler

Proposed Position Source Monitor and Source Type Subfields:

The following subparagraphs provide proposed material to add Position Monitor and Position Source Type subfields to the Aircraft Operational Status Message as provided in the following MOPs paragraphs. Note the changes or new text are provided in "blue" font. Existing MOPs text is retained in "black" font. Particular points for discussion are in "red" font.

2.2.3.2.7.2.4 "OPERATIONAL MODE (OM)" Subfield in Aircraft Operational Status Messages

The "Operational Mode (OM)" subfield is a 16-bit subfield ("ME" bits 25 through 40, Message bits 57 through 72) that **shall** indicate Operational Modes that are active on board the A/V in which the ADS-B Transmitting Subsystem resides. The format of the OM subfield in Aircraft Operational Status Messages **shall** be as specified in Table 2-68.

Table 2-68: Operational Mode (OM) Subfield Format

Msg Bit #	57	58	59	60	61	62	63	64---67	68---72
"ME" Bit #	25	26	27	28	29	30	31	32---35	36---40
OM Format	0 0		RA Active	IDENT Switch Active	Reserved for Receiving ATC Services	Single Antenna Flag	Position Monitor Active	Position Source Type	Reserved
	0 1								
	1 0								
	1 1								

2.2.3.2.7.2.4.5 "Position Monitor Active" OM Subfield in Aircraft Operational Status Message

The "Position Monitor Active" Operational Mode code is a one-bit subfield ("ME" bit 31, message bit 63) of the OM Code subfield in Aircraft Operational Status Messages.

a. The ADS-B Transmitting Subsystem **shall** set the "Position Monitor Active" to ONE (1) whenever the position Source that is declared in the "Position Source Type" subfield (see 2.2.3.2.7.2.4.6) is actively being processed by a position source monitor that is selecting the best possible source or

providing adjusted position data based on multiple position sources being available to the monitor.

Note: *It is the responsibility of the Position Source Monitor to provide appropriate indication to the ADS-B Transmitting Subsystem whenever it is actively monitoring multiple sources to establish the position information provided to the ADS-B Transmitting Subsystem. As an example, an FMS System using multiple IRS systems to establish the final position data would need to provide an appropriate indication such that the ADS-B Transmitting Subsystem could set the "Position Monitor Active" to ONE (1).*

*The requirement for an interface by which the ADS-B Transmitting Subsystem may accept an indication of Position Source Monitor status is given in section 2.2.5.1.**TBD.***

b. The ADS-B Transmitting Subsystem **shall** set the "Position Monitor Active" to ZERO (0) whenever the Position Source is NOT actively being processed as declared by the monitor or there is no monitor data provided to the ADS-B Transmitting Subsystem.

Discussion point: Should the NIC be set to zero if "Position Monitor Active" is set to ZERO?

2.2.3.2.7.2.4.6 "Position Source Type" OM Subfield in Aircraft Operational Status Message

The "Position Source Type" Operational Mode code is a 4-bit subfield ("ME" bits 32 through 35, message bits 64 through 67) of the OM Code subfield in Aircraft Operational Status Messages. The "Position Source Type subfield is used to indicate the type of Position source being used to provide Position data to the ADS-B Transmitting Subsystem and **shall** be encoded in accordance with Table 2-69.

Table 2-69: Position Source Type Operational Mode (OM) Subfield Encoding

Coding (Binary) (32---35)	Coding (Decimal)	Meaning	Notes
0000	0	No Position Source Type information is available or the Source Type is UNKNOWN	1
0001	1	GPS	1
0010	2	GPS/IRS Hybrid	3
0011	3	FMS_ARINC 702A Supplement 3 Compliant	1,2
0100	4	FMS_Other than ARINC 702A Supplement 3 Compliant	3
0101	5	DME/DME	3
0110	6	DME/IRS	3
0111	7	IRS Coast Mode	3
1000	8	IRS	1,3
1001	9	IRS/IRS Voted	3
***	***	RESERVED	
***	***		
***	***		
1111	15		

Notes:

1. It should be feasible to establish these encoding based on what information is directly provided to the ADS-B Transmitting Subsystem or what systems are directly connected to the ADS-B Transmitting Subsystem.
2. ARINC 702A Supplement 3 Compliant means that the FMS is providing Position, Velocity, Time (PVT) information to the ADS-B Transmitting Subsystem in accordance with ARINC 702A Supplement 3. Data must include ANP and RNP data.
3. These encodings should be established based on information provided to the ADS-B Transmitting Subsystem by the FMS or other system that is determining the Position source type. As such, these systems must be modified to provide indication of the Position source type to the ADS-B Transmitting Subsystem.
4. The requirement for an interface by which the ADS-B Transmitting Subsystem is advised of the Position Source Type is given in section 2.2.5.1.**TBD**.
5. Corresponding to Discussion Point above: Although the transmitted NIC will be equal to ZERO for some position sources, knowledge of the position source may enable an independent determination of the integrity of the position (i.e., the NIC).

2.2.3.2.7.2.4.7 "Reserved" OM Subfield in Aircraft Operational Status Message

"ME" bits 36 through 40, Message bits 68 through 72 are Reserved for future assignment as may be needed in these or future MOPs.

RHS_Commentary:

Note that once the above is agreed and formalized, appropriate interface requirements sections will need to be added to section 2.2.5.

ATTACHMENT E

CONTAINMENT RADIUS PROBABILITY FOR NON-GNSS SENSORS

Honeywell International Inc.

Background

The containment radius for GNSS sensors (i.e. Horizontal Protection Limit or HPL) is defined such that the probability of exceeding this radius without an alert is $1.0\text{x}10^{-7}$/hr. This probability rate is arrived at by assuming that a satellite in view fails randomly at the rate of $1.0\text{x}10^{-4}$/hr. This random failure rate combined with $1.0\text{x}10^{-3}$ missed-detection probability yields the overall undetected failure rate $\lambda = 1.0 \times 10^{-7} / \text{hr}$. Based on this failure rate, the probability of exceeding the containment radius over some time interval T can be computed as follows

$$P(T) = 1 - e^{-\lambda T} \tag{1}$$

For a non-GNSS sensor such as an inertial navigation system, the containment radius is based on rare-normal excursions of the position solution – not on failure rates. These excursions are driven by normal sensor errors with Gaussian statistics. In the case on an inertial navigation system, at any instant of time the probability density function of the radial position error can be overbound by a Rayleigh distribution. Thus, a conservative estimate of the probability of exceeding the containment radius at that instant p_{sample} can easily be determined. However, because the position is computed through a process of integrating angular rates, attitudes, accelerations, and velocity, the position error at any one instant is correlated to previous values. The degree of correlation generally decreases with increasing time difference. Because successive samples of the position error are not independent, computing the probability of exceeding the containment bound over some time interval is not trivial. However, through simulation one could determine an effective time Δt_{ind} between "independent" samples. The effective rate of exceeding the containment radius is then

$$\lambda_{eff} = \frac{p_{sample}}{\Delta t_{ind}} \tag{2}$$

Where Δt_{ind} is in hours. This rate is an approximation and is not valid for time intervals less than the Δt_{ind}. For example, for a containment probability of 0.95, the instantaneous probability that a sample exceeds the containment bound is $p_{sample} = 0.05$. If the effective time between independent samples is 0.1 hours, then the effective rate of exceeding the containment radius is

$$\lambda_{eff} = \frac{p_{sample}}{\Delta t_{ind}} = \frac{0.05}{0.1 \text{ hr}} = 0.5/\text{hr} \tag{3}$$

Based on the effective rate, probability of exceeding containment radius over a time of $T = 1$ hour is

$$P(T) = 1 - e^{-\lambda T} = 1 - e^{-(0.5)(1.0)} \cong 0.393 \tag{4}$$

Based on discrete probability, we have 10 "independent" samples over the 1 hour period. The probability of exceeding the radius at least once in that hour is the complement of the probability of not exceeding it during that hour or

$$P(T) = 1 - \left(1 - p_{sample}\right)^{10} = 1 - \left(1 - 0.05\right)^{10} \cong 0.401 \tag{5}$$

These two methods yield nearly the same result. But, now let $T = 0.01$ hour . Based on the effective rate, the probability is now

$$P(T) = 1 - e^{-\lambda T} = 1 - e^{-(0.5)(0.01)} \cong 0.0050 \tag{6}$$

But, based on the discrete probability we have only one independent sample, so

$$P(T) = 1 - \left(1 - p_{sample}\right) = 1 - \left(1 - 0.05\right) \cong 0.05 \tag{7}$$

These two methods now differ by an order of magnitude. This confirms the assertion that the effective rate method is not valid for times less than the time between independent samples.

From this exercise, it is also clear that the user should have knowledge of the time between "independent" samples when computing the probability of exceeding a containment bound over some time interval of interest for a non-GNSS sensor whose probability is inherently instantaneous.