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<u>RTCA Special Committee 186, Working Group 3</u>

<u>ADS-B 1090 MOPS, Revision A</u>

<u>Meeting #17</u>

Supporting Technical Material

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SUMMARY

This paper documents some of the detailed analysis that was done at Lincoln in the process of generating the performance results in Appendix P. No action is required of WG-3.

Supporting Technical Material

This paper documents some of the detailed analysis that was done at Lincoln in the process of generating the performance results in Appendix P. While developing the simulation tools, and while making the runs for Appendix P, we made a large number of checks for consistency and supporting analyses, which are not documented in the appendix. Some of the key supporting material is documented in this paper.

TOP-BOTTOM TRANSMITTING DIVERSITY

As documented in section P.3.2.1, the way that the track-level simulation takes account of the top-bottom antenna diversity for the transmitting aircraft is to take the average of the two values of reception probability.

$$P(av) = 0.5 * P(top) + 0.5 * P(bot)$$

After the average is computed, it is used as if all receptions had this probability. This is a simplification, justified in the Appendix P text by saying simply that each antenna transmits 50% of the messages.

That explanation is also a simplification. We have made a more detailed analysis of this issue, summarized as follows. Given that the two reception probabilities may be unequal, the probability of receiving one or more squitters in 12 seconds is

$$P(exact) = 1 - (1 - P(top))^{24} * (1 - P(bot))^{24}$$

The simplified calculation is

$$P(approx) = 1 - (1 - P(av))^{48}$$

To compare these two formulas, Figure 1 was constructed, which shows combinations of two probability values that achieve a total reception probability of 0.95. The open circles are combinations of p1 and p2 for which P(exact) = 0.95. The solid line shows combination of p1 and p2 for which P(approx) = 0.95.

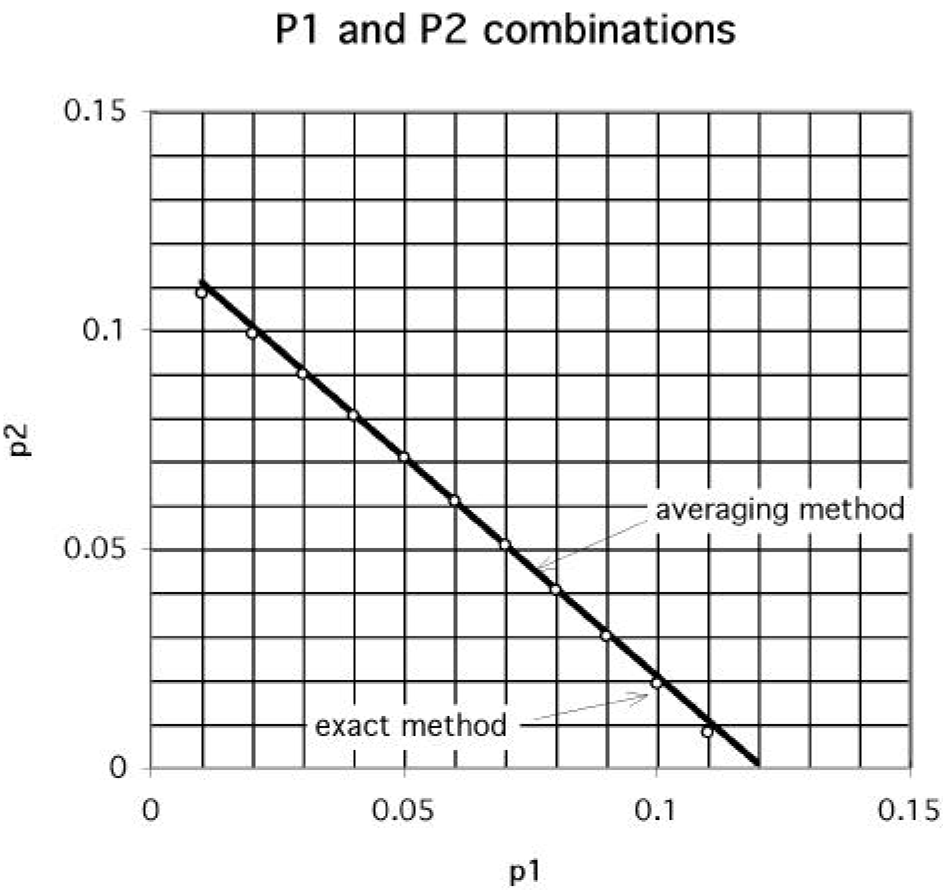

Figure 1. Comparison of two formulas.

The results in the figure make it clear that the two formulas have nearly the same effect. Furthermore, the slight difference is in the direction which makes the calculated performance slightly worse using the averaging method. Therefore the averaging method is an accurate method of combining the top and bottom transmitting antennas in the track-level simulation.

EFFECT OF AIRCRAFT MOTION

We spent a lot of time discussing a difference between the Lincoln simulation and that of APL regarding aircraft motion. In the Lincoln simulation, when the aircraft antenna gains are selected at random in the beginning of a run, they are kept constant. Therefore motion of the aircraft has no effect. In the APL simulation, the aircraft move for a duration of 300 seconds, and during that time it is possible for some of the antenna gains to change. Whether a particular antenna gain changes or not depends on whether there is much change in the azimuth angle in the direction of the other aircraft. In most cases these azimuths don't change much in 300 seconds, so normally the antenna gains remain constant. But given that this is a difference in the two evaluations, we spent time determining whether one method would yield higher or lower performance than the other.

We formulated a comparison as follows, which was Larry's idea. Imagine that a 300 second period is divided into two halves, 150 seconds each. And for simplicity, imagine that the antenna gains change at the midpoint, being selected randomly again and independently of the original selection.

Beginning with Lincoln's fixed-gain technique, Figure 2 illustrates the connection between received signal power and the MASPS requirement. The upper plot shows, for a pair of aircraft, the probability of successfully receiving an update in 12 seconds, as a function of received power S. Of course, this probability increases monotonically with S, and reaches 0.95 at some point. At that point and above, the update-rate requirement is met. Weaker signals are possible, and the MASPS is satisfied if these do no occur more frequently than 5 percent of aircraft pairs (the shaded area in the lower plot). This lower plot applies at a particular range, and if the range were increased, the bell shaped distribution would move to the left, and the shaded area would increase. If range is increased to the point at which the shaded area is exactly 5%, that is the maximum range satisfying the MASPS.

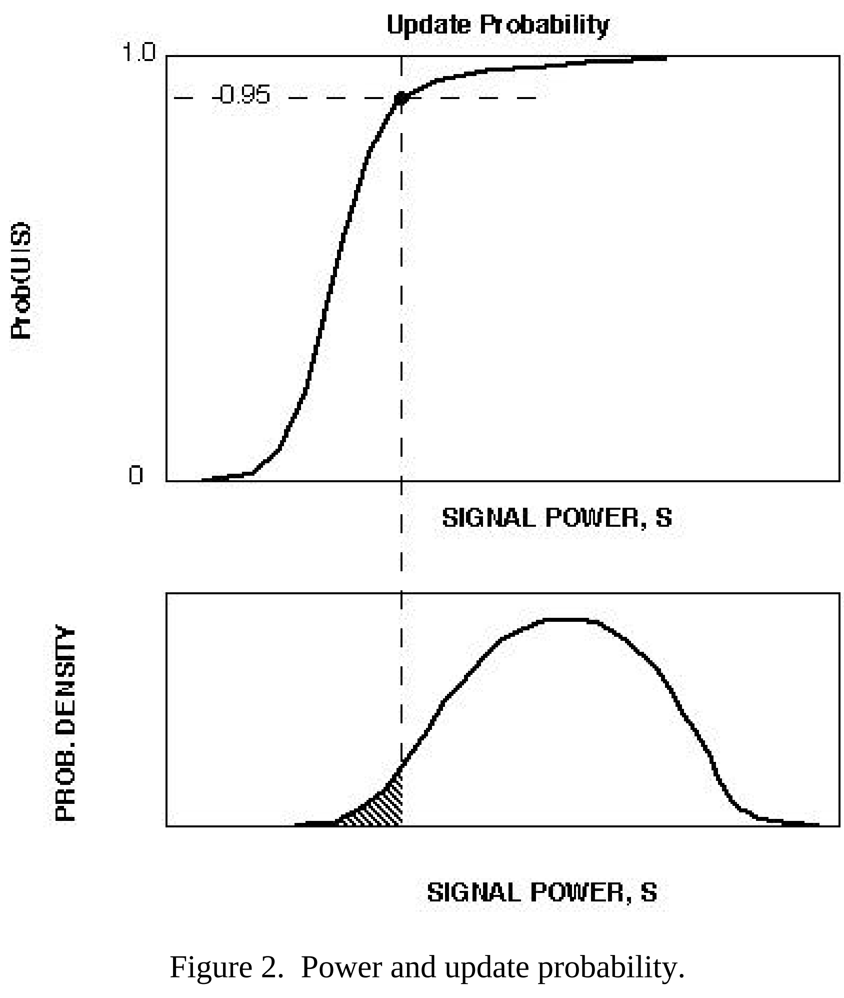

Figure 2. Power and update probability.

Although signal power S is a random variable, the function Prob(U|S) is a known function. Therefore we can define

$$x = \text{Prob}(U|S)$$

to be a function of a random variable, and therefore x is a random variable. The probability distribution of x is approximately as shown below for a value of range near the max. satisfying the MASPS. The mean value of x is much higher than 0.95. In fact, because the function is monotonic, the tail area, below 0.95 is the same as the shaded area above.

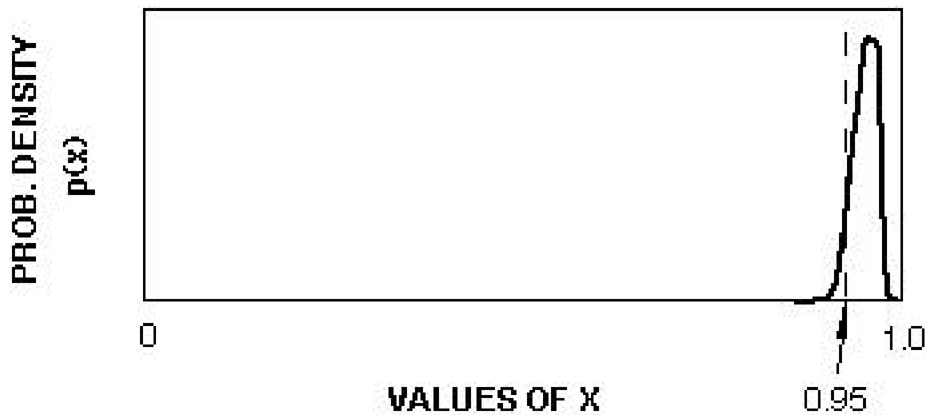

Figure 3. Probability distribution of x.

These illustrations apply to the Lincoln formulation, in which antenna gains are constant. Consider the time varying case, in which received power is selected randomly at t = 0 and selected again randomly and independently at t = 150 sec. During the first half, x can be defined as above. During the second half a new value of x occurs, so we can call these x1 and x2. If we chose an observation time at random over the 300 seconds, the probability distribution of x would be

$$p(x) = 0.5 * p(x1) + 0.5 * p(x2)$$

Note that the two values of x have the same probability distribution (of the character illustrated in Figure 2), and they are independent. Therefore the mean of p(x) is the same as the mean of p(x1) which is the same as the mean of p(x2). But the standard deviation is smaller, by a factor of the square root of 2.

$$\text{sigma}(x) = \text{sigma}(x1)/\text{sqr}(2)$$

Therefore the distribution of the combined x is made tighter by the change of antenna gains after 150 seconds. Therefore, if range were selected to make the tail area = 5% in the upper figure (in Figure 2), then the tail area for the combined x, for the time varying formulation, would be smaller. The performance would be better than the MASPS requirement.

This answers the original question. It shows that to allow antenna gains to vary tends to make the calculated performance better that it would be if antenna gains were held constant.

We believe the actual change in calculated performance would be very slight. Larry ran some comparison cases to check this, and the results indicate that there was no significant difference in performance resulting from the changing of antenna gains through aircraft motion.