1090-WP-16-02R1
12 December 2002

RTCA Special Committee 186, Working Group 3

ADS-B 1090 MOPS, Revision A

Meeting #16

1090 MHz Fruit Models and Reception Probability Models

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SUMMARY
This Working Paper reviews the characteristics of three arrival process models (referred to here as Bernoulli, Poisson, and Pascal) for 1090 MHz Fruit, and describes three decode probability models (referred to here as Geometric, Linear, and Quadratic) for 1090 MHz Extended Squitter (ES).

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INTRODUCTION

This working paper reviews the characteristics of three arrival process models (referred to here as Bernoulli, Poisson, and Pascal) for 1090 MHz Fruit, and describes three decode probability models (referred to here as Geometric, Linear, and Quadratic) for 1090 MHz Extended Squitter (ES), in an attempt to answer three questions:

1. How to determine if the 1090 MHz ATCRBS Fruit arrival count data fits a Negative Binomial Distribution (NBD)?
2. How may a NBD Fruit environment effect 1090 MHz ES reception performance?
3. How to generate with hardware, and/or simulate with software, Fruit arrival data with an appropriate inter-arrival time distribution?

BACKGROUND

Re/Q1. WP-15-01 proposed using the method of moments (Ref.1 and Fig.5) to estimate the parameters of the NBD, and then using it to fit the over dispersed 1090 MHz ATCRBS Fruit arrival count (frequency of overlaps) data, reported in SC-186 WG-3 working papers WP-12-14, WP-13-05, WP-13-14, and WP-14-12. A preliminary comparison was made for some of the LA airborne Fruit data, which had a variance-to-mean ratio between 2 and 2.5, only for Fruit overlaps (time interval occupancy numbers) of zero, one, and two. This working paper extends this analysis, using NBD table look-up, to eight overlaps.

Re/Q2. WP-14-15 reported on the results of a simulation of the effect of non-Poisson ATCRBS Fruit on ES reception performance, using a curve fit to some of the LA airborne cumulative Fruit inter-arrival time data, for the high power Fruit. It was noted that the total (high, and low power Poisson) pulse-level simulated Fruit applied to the receiver had a variance-to-mean ratio of 1.6. The main conclusion of the analysis was "that the non-Poisson timing behavior of ATCRBS fruit does not have a significant effect on Extended Squitter reception probability", which was presented as a function of the received signal power (dBm at antenna).

This working paper examines the effect of Poisson and NBD ATCRBS Fruit on ES reception performance; using different models (linear and quadratic curve fits) of decode probability (a function of the number of ATCRBS interferers, referred to as reception probability (average over received signal power centered at the average fruit power), in WP-10-08) for Appendix I and LDPU receivers. The decode probability is summed (or integrated) over the terms of the Fruit distribution to obtain the reception probability. (You might say that this reception probability measures the integral effect of the Fruit distribution on reception performance, whereas, the reception probability in WP-14-15, and the 1090 MHz Extended Squitter Assessment Report, June 2002 (WP-12-05), measures the differential effect of the received power on reception performance. It is proposed that small differences in probability, spread out over the differential curve, will show up in the integral, provided it is properly normalized.)

Re/Q3. WP-15-01R contains (3) figures. I have completed (by definition) the analysis (best guess) of this question, and have modified and expanded the number of figures (10).

DISCUSSION & FIGURES 1-5

It is widely accepted that a Poisson distribution in counts (number/time interval), is consistent with exponential inter-arrival, and gamma arrival separation (waiting time for the nth count) distributions in the time domain. See Row 3 of Fig.1. Less well appreciated is the fact that a BD in counts is consistent with geometric (GD-0) interarrival, and negative binomial (NBD-0) arrival separation distributions. See Row 2 of Fig.1. (Also see Ref.4.) The problem faced in this part of the study was, if you have a modified form of the NBD-2 (a NBD postulated to exist that reduces to the PD for small a priori probability, to be discussed in Fig.2) in counts, what are the interarrival and arrival separation distributions? This analysis indicates that modified forms of the geometric (GD-1) interarrival, and negative binomial (NBD-1) arrival separation distributions, are consistent with a NBD-2 count distribution. See Row 4 of Fig.1.

Fig.1: Arrival Process Distributions. Original figure (found in WP-15-01R, presented to WG-3 at meeting #15), with headings of Columns 2, 3, and 4, modified to hopefully make them clearer, and best “guesses” for “missing” distributions added to columns 3 and 4. There doesn’t appear to be any universally accepted naming of processes and/or distributions, which adds to the general confusion of the subject. For example, what I’m calling the Pascal process may also go by other names (Polya process, Gamma-Mixed Poisson process, etc.), because of the many ways that the NBD can be mathematically generated (Refs.2 and 3), and it is also a Bernoulli-type (p+q=1) process. The modified geometric distribution (GD-1) and/or the negative binomial distribution (NBD) are also some times referred to as the Pascal distribution. The NBD may be more heavily used in the “real” world (versus the classroom) than its’ relative, the BD, which may be a reflection of the fact that the NBD appears in 3 forms in Fig.1.

1. Column 1: F-D = Fermi-Dirac; M-B = Maxwell-Boltzmann; B-E = Bose-Einstein.
2. Columns 2, 3, and 4: Details for each column starting in Figs.2, 7 and 10, respectively. In the limit (see Fig.2 and Fig.7 notes): BD and NBD-2 reduce to Poisson, GD-0 and GD-1 reduce to Exponential, and NBD-0 and NBD-1 reduce to Gamma.
3. 3 NBD’s are defined in Figs.2 and 10. NBD-1 and NBD-2 are the 2 forms of the NBD described in WP-15-01.
4. Column 3: Why I am calling X = GD-1 will become clear in Fig. 9, where the 2 geometric distributions (GD-0 and GD-1) are defined.

Fig.2 (New): Arrival Count Distribution. To answer Jim's question, everything in Figs.2 thru 5 is dimensionless. However, there is another caution associated with these figures. I have used the same symbols for the parameters in the three distributions. In fitting data, for comparison purposes, it may be useful to assume that the parameters have the same meanings/definitions for the three distributions. However, in calculating a distribution, this may not be a good assumption. I assumed they were different in my previous Navy study (see background section of WP-15-01). There wasn't time to complete a study of this problem.

1. Column 2: $f(x)$, normalized to one (1), is the probability distribution for the discrete random variable x = the number of arrivals (in a time interval or bin). Note differences/similarities in domain of x.
2. Columns 2 and 3: There are two parameters, n and p, for the BD; one parameter, M for the PD, which may be set equal to np; and two parameters, n and $q=1-p$, for the NBD-2.
3. Columns 2, 3, and 4: Note similarities/differences
4. Column 4: Three cases: Binomial $M>V$; Poisson $M=V$; Negative Binomial $M<V$. In the limit, if $p<<1$ ($q\sim 1$), then all three distributions are approximately the same.

Fig.3 (New): Reception Probability Models I. Figs. 3-6 relate to Fruit and the signal.

1. Column 1: Fruit arrival model.
2. Column 2: $f(0)$ = Probability of no fruit (non-arrival probability), which in the "old days" was the signal Reception Probability. Note that $B(0)$ and $NB(0)$ are the same if n and q are defined the same. Note $(1-p)^n < \exp(-np)$.
3. Column 3: Sometimes, if a signal could be decoded with 1 fruit present, then the reception probability would be $f(0)$ plus some fraction of $f(1)$. Assume, the contribution from $f(1)$ to the reception probability is $(1-k)f(1)$, where k is the fraction of $f(1)$ lost. As noted, in the "old days", k might equal 1, or .6, or .3, for example, depending on many factors including the fruit and signal power levels. With enhanced decoding, k may now equal .01, for example.
4. Column 4: Linear decoder model. In addition, higher terms, for example, $f(2)$, $f(3)$, $f(4)$, etc., may now contribute to the overall reception probability with enhanced decoding. The linear decoder model accounts for this, by letting the fraction of the contribution from each number of fruit overlaps be equal to $(1-kx)$, where x is the number of fruit overlaps, and k is a constant. In other words, the linear reception probability is given by: $f(0) + (1-k)f(1) + ((1-2k)f(2) + \ldots$ Note, if all the parameters are the same for the three distributions, then the NBD Reception Probability is lowest. However, in terms of their respective means, the three fruit models give the same Reception Probability for the linear decode model. Using a linear decoder would support the results in WP-14-15 that reception performance does not change significantly with Fruit model. However, what if it's not a linear decoder?

Fig.4 (New): Reception Probability Models II. There is a double caution associated with this figure. In addition to using the same symbols for the parameters in the Fruit distributions, I have also used the same symbol k for the parameter in the decode probability distribution. For comparison purposes this may be OK, but when you're fitting a set of data, obviously the values of k will be different depending on which decode model you are using.

1. Column 1: Fruit arrival model.
2. Column 2: Geometric Decoder Model (A best case scenario for NBD). The geometric (series) decoder model accounts for decoding of higher terms, by letting the fraction of the contribution from each number of fruit overlaps be equal to $(1-k)^x$, where x is the number of fruit overlaps, and k is a constant. In other words, the geometric reception probability is given by: $f(0) + (1-k)f(1) + ((1-k)^2)f(2) + \ldots$ It appears that for a given k, the NBD fruit model gives the largest Reception Probability for the geometric decode model. The geometric decoder model is concave up, similar to an exponential (more about that later).
3. Column 3: Linear Decoder Model repeated for comparison.
4. Column 4: Quadratic Decoder Model (A worst case scenario for NBD). I looked at WP-10-08, where there is a figure showing both simulation (using Appendix I, 4-4 table) and LDPU reception probability versus the number of ATCRBS interferers. The curves in this figure represent what I am calling decode probability, and they appear concave down. One of the simplest concave down models would be the quadratic model. The quadratic decoder model accounts for decoding of higher terms, by letting the fraction of the contribution from each number of fruit overlaps be equal to $(1-k(x^2))$, where x is the number of fruit overlaps, and k is a constant. In other words, the quadratic reception probability is given by: $f(0) + (1-k)f(1) + ((1-4k)f(2) + \ldots$ It appears that for a given k, the NBD fruit model gives the smallest Reception Probability for the quadratic decode model. To see this more clearly, see Fig.5, Column 4 note.

Fig.5 (New): Method of Moments. Problem: From the point of view of the data, don't know n, and p or q. Therefore, I will use the method of moments (mean and variance) technique (Ref.1) to estimate n, and p or q. (Other methods exist, e.g., maximum likelihood estimation (MLE), but they are more complicated and do not lead to closed form solutions.)

1. Column 1: See Fig.2 Column 4 for true (population) values of M and V.
2. Columns 2 and 3: Using the measured (sample) values of M and V in the two moment relationships given for each distribution in Fig.2 Column 4, solved for q and n in terms of M and V. Measured values of M and V are used in the Numerical Results section to estimate the values of the first eight (8) terms of the NBD for LA airborne data.
3. Column 4: These expressions derived from the fact that the variance of a probability law is equal to its mean square, minus its square mean. Measured values of M and V are used in the Numerical Results section to estimate the Reception Probability for a Quadratic $(1-k(x^2))$ Decoder, and it is compared to the results for a Linear $(1-kx)$ Decoder where only M is required. For data with a given M: BD has largest quadratic decode Reception Probability, since $V<M$; and NBD has the smallest quadratic decode Reception Probability, since $V>M$.

NUMERICAL RESULTS

1. LA Data Compared to NBD Table Look-Up. The same LA airborne data (contained in memos dated 31 July 02, and 2 August 02, from Bill Harman, and WP-13-14) compared in WP-15-01, but now for occupancy numbers zero (0) thru eight (8), were compared to table look-up values for the NBD found in Ref.5 (page numbers given below). For the LA data the values (estimates) of q and n are based on the sample values of M and V (Method of Moments): whereas in the Table, the population (true) values of M and V are based on the values of q and n. Keeping in mind that the table look-up values were not interpolated, and that the measured values were read from histograms, the results below appear acceptable. While there may be something else going on in the low occupancy bins, the tail of the distribution seems to be well matched.

LA data (Higher Threshold):

NBD	q	n	Mean	Variance
LA data fit	0.48	1.51	1.6	3.3
Table (p. 159)	0.48	1.5	1.63	3.39

x fruit	0	1	2	3	4	5	6	7	8
Data	.32	.28	.18	.10	.06	.03	.02	.01	.01
Table	.33	.26	.17	.10	.06	.03	.02	.01	.01

LA data (Normal Threshold, -84 dBm at antenna):

NBD	q	n	Mean	Variance
LA data fit	0.45	1.57	1.96	4.4
Table (p. 146)	0.44	1.6	2.04	4.68

x fruit	0	1	2	3	4	5	6	7	8
Data	.24	.28	.21	.12	.07	.04	.03	.02	.01
Table	.27	.24	.18	.12	.08	.05	.03	.02	.01

LA data (Lower Threshold):

NBD	q	n	Mean	Variance
LA data fit	0.42	2.06	2.8	6.6
Table (p. 139)	0.42	2.0	2.76	6.58

x fruit	0	1	2	3	4	5	6	7	8
Data	.13	.22	.21	.15	.10	.07	.04	.03	.02
Table	.18	.20	.18	.14	.10	.07	.05	.03	.02

2. Receiver Performance in a NBD Fruit Environment. This is an estimate of the 1090 MHz ES reception probability in a NBD fruit environment, based on linear and quadratic fits to the LDPU and Lincoln Laboratory simulated (based on techniques in Appendix I) data presented in WP-10-08. Geometric model also included in Fig.4, but does not fit this data. The models do not explicitly account for fruit and signal power levels, other than their effect on the decode parameter k.

Linear Decode Model: See Fig.4 Column 5 note, which refers to WP-10-08, which contains a figure showing both simulation (using Appendix I, 4-4 table) and LDPU reception probability versus the number of ATCRBS interferers. The curves in this figure represent what I am calling decode probability. I will attempt to fit lines to these curves. The table below contains numerical values for the Linear (1-kx) Decode model. The Appendix I (4-4 table) curve very approximately fits the k = .04 line, and the LDPU very approximately fits the k = .08 line. From this we can determine the Reception Probability in the next table, given the mean fruit rate (with a decode interval = 100 microseconds). The Reception Probability decreases with Fruit rate, as expected.

Linear Decode Probability = 1 – kx

k x fruit	1	2	3	4	5
.02	.98	.96	.94	.92	.90
.04 (App. I)	.96	.92	.88	.84	.80
.06	.94	.88	.82	.76	.70
.08 (LDPU)	.92	.84	.76	.68	.60
.10	.90	.80	.70	.60	.50

Linear Decode Reception Probability = 1-kM (M=100 microsecond bins*fruit rate)

k fruit rate	10,000/sec.	15,000/sec.	20,000/sec.	25,000/sec.	30,000/sec.
.02	.98	.97	.96	.95	.94
.04 (App. I)	.96	.94	.92	.90	.88
.06	.94	.91	.88	.85	.82
.08 (LDPU)	.92	.88	.84	.80	.76
.10	.90	.85	.80	.75	.70

Quadratic Decode Model: See Fig.4 Column 5 note, which refers to WP-10-08, which contains a figure showing both simulation (using Appendix I, 4-4 table) and LDPU reception probability versus the number of ATCRBS interferers. The curves in this figure represent what I am calling decode probability, and they appear concave down. I will attempt to fit a quadratic to these curves as being one of the simpler concave down models. The table below contains numerical values for the Quadratic (1-k(x^2)) Decode model. The Appendix I (4-4 table) curve very approximately fits the k = .010 curve, and the LDPU very approximately fits the k = .020 curve. From this we can determine the Reception Probability in the next set of tables, given the mean fruit rate (with a decode interval = 100 microseconds), and the Variance/Mean (V/M) ratio of the NBD. The Reception Probability decreases with Fruit rate, as expected, and it also decreases with V/M ratio.

Quadratic Decode Probability = 1 – k(x^2)

k x fruit	1	2	3	4	5
.005	.995	.980	.955	.920	.875
.010 App. I	.990	.960	.910	.840	.750
.015	.985	.940	.865	.760	.625
.020 LDPU	.980	.920	.820	.680	.500
.025	.975	.900	.775	.600	.375

Quadratic Decode Reception Probability = 1-k((M^2)+V) (M=1=100 microsecond bins * fruit rate of 10,000 fruit/sec.) as a function of fruit variance (V).

k	V=M=1	V=1.5M	V=2M	V=2.5M	V=3M
.005	.9900	.9875	.9850	.9825	.9800
.010 App. I	.9800	.9750	.9700	.9650	.9600
.015	.9700	.9625	.9550	.9475	.9525
.020 LDPU	.9600	.9500	.9400	.9300	.9200
.025	.9500	.9375	.9250	.9125	.9000

Quadratic Decode Reception Probability = 1-k((M^2)+V) (M=2=100 microsecond bins * fruit rate of 20,000 fruit/sec.) as a function of fruit variance (V).

k	V=M=2	V=1.5M	V=2M	V=2.5M	V=3M
.005	.9700	.9650	.9600	.9550	.9500
.010 App. I	.9400	.9300	.9200	.9100	.9000
.015	.9100	.8950	.8800	.8650	.8500
.020 LDPU	.8800	.8600	.8400	.8200	.8000
.025	.8500	.8250	.8000	.7750	.7500

Quadratic Decode Reception Probability = 1-k((M^2)+V) (M=3=100 microsecond bins * fruit rate of 30,000 fruit/sec.) as a function of fruit variance (V).

k	V=M=3	V=1.5M	V=2M	V=2.5M	V=3M
.005	.9400	.9325	.9250	.9175	.9100
.010 App. I	.8800	.8650	.8500	.8350	.8200
.015	.8200	.7975	.7750	.7525	.7300
.020 LDPU	.7600	.7300	.7000	.6700	.6400
.025	.7000	.6625	.6250	.5875	.5500

Summary of Results for Linear and Quadratic Decode Models:

Decode Model:	Linear	Quadratic	Quadratic	Quadratic
10K/sec Fruit Environment:	Any	Poisson (V/M=1)	NBD (V/M=2)	Diff.
Appendix I (4-4 table):	.96	.98	.97	.01
LDPU:	.92	.96	.94	.02
20K/sec Fruit Environment				
Appendix I (4-4 table):	.92	.94	.92	.02
LDPU:	.84	.88	.84	.04
30K/sec Fruit Environment				
Appendix I (4-4 table):	.88	.88	.85	.03
LDPU:	.76	.76	.70	.06

This is the way I would interpret the results for an over-dispersed (e.g., V/M=2) NBD Fruit environment. If the reception probability were the same for linear and quadratic decode models at a specified fruit rate (here 20K/sec.), then I would expect that at lower fruit rates (e.g., 10K/sec.) the quadratic decode model would do better, and at higher fruit rates (e.g., 30K/sec.) the linear decode model would do better. Graphically, this makes sense.

DISCUSSION & FIGURES 6-10

In Figs.6-10, time enters the picture through the relationship M = at, where M is the (measured or true) mean number of arrivals, a is the (measured or true) mean Fruit rate (units of inverse time), and t is the time interval. Actually, t is used to define the p's (or q's), and should have been discussed before Figs.1-5, as a motivation for the way the distributions may be associated with the counting statistics (Refs.6 and 7).

Fig.6: Reception Probability Estimates. This figure is a Summary of Results for Linear and Quadratic Decode Models also found in the Numerical Results Section.

Fig.7: Inter-arrival Separation. This is the original fig.2 (found in WP-15-01R, presented to WG-3 at meeting #15), with the following changes: In Column 1 show how f(0;n,p) becomes f(0;n,M); changed f's to g's in headings to Columns 3 and 4, to make it clear that they are different functions; let x stand for the discrete random variable in Column 4, and filled in a name for distribution X.

1. Column 1: Note that in the limit, if n>>1, then all three distributions are the same. Compare with Fig.2.
2. Column 2 is the same as Column 1, with M = at, where a is the true mean fruit rate in a time interval of length t. This term is set equal to 1 – cumulative probability P (expressed as a function of the time interval).
3. Column 3: g(t) is the derivative of the cumulative probability with respect to the time interval. It is normalized to one (1)(?), and is the probability distribution for the continuous random variable t = the time between two arrivals (in a time interval or bin). There are two parameters, n and a, for the distributions associated with the Bernoulli and Pascal processes, and one parameter, a, for the distribution associated with the Poisson process. I have indicated "best guess" names for these distributions. For the Binomial, it appears to be a generalized Beta distribution. For the Poisson, it is the well-accepted Exponential distribution. And for the Negative Binomial, it appears to be a generalized Pareto distribution.
4. Column 4 is a discrete version of Column 3, setting t=n=x. (Using x so as not to get confused about variable vs. parameter.) I still have questions about this step, and the others in this figure. The result in Row 2, i.e., that the GD-0 is the interarrival distribution for the BD, is generally accepted. Therefore, I shall <u>assume</u> that the result in Row 4 is correct, i.e., that the GD-1 (see Fig.9 for justification of name) is the interarrival distribution for the NBD-2. Note: To normalize GD-1, it is required that the domain of x include zero (0).

Fig.8: Inter-arrival Time. Same as original (found in WP-15-01R, presented to WG-3 at meeting #15), except for: the addition in Column 1 of the values of a as required when t=n=z in Column 4 of Fig.7; and the name of X. In Fig.7, for the Binomial, M=np=at, therefore a = p; and for the Negative Binomial-2, M=(np/q)=at, therefore a=p/q.

Fig.9 (New): Geometric Distribution (GD)

1. Column 1: Geometric (GD-0), and modified geometric (GD-1) distributions. Note domain of x.
2. Column 2: Same as Column 4 of Fig.7, with a as specified in Fig.8.
3. Rows 2 and 4: The difference between the geometric (GD-0) and the modified geometric distribution (GD-1), with the same parameter, is the way in which they count: GD-0 starts at one, and GD-1 starts at zero.

Fig.10 (New): Arrival Separation Distribution. It is noted from the distributions, means, and variances, given in Fig.9 and Fig.10, that the NBD's are the sum of n GD's, or, to put it another way, the GD is a special case of the NBD with n=1.

1. Row 2: NBD-0.
2. Row 3: Gamma Distribution.
3. Row 4: Besides an interchange of parameter p and q, there is no difference between NBD-1 and NBD-2 found in Fig.2.

PRELIMINARY CONCLUSIONS

Re/Q1: See WP-15-01, and in this working paper, Numerical Results Section, LA Data Compared to NBD Table Look-Up. The LA airborne Fruit data appears to fit to the NBD (actually better at the higher number of Fruit overlaps).

Re/Q2: See Numerical Results Section, Receiver Performance in a NBD Fruit Environment, Summary of Linear and Quadratic Decode Models. It depends on the decode probability. Linear decode, maybe no effect. But, quadratic decode reception probability would depend on the characteristics (i.e., inversely depend on the V/M ratio) of the NBD Fruit environment, and may be expected to be degraded (by a few percent) in performance compared to a Poisson Fruit environment, as the Fruit rate increases. The decode models in this working paper are based on a limited set of data – zero to five Fruit overlaps. Higher terms will be necessary to determine the actual shape of the probability curve. Even a combination of the three decode models presented in this working paper is possible, e.g., quadratic for V/M ~ 1, linear for V ~ 2, and geometric for V/M > 3.

Re/Q3: See Figs.7-10.

1. This is still a sketchy analysis.
2. With regard to continuous time domain distributions, it seems that a particular form of a continuous generalized Pareto distribution (parameters n and a, see Fig.7) in the time domain, may be the best "bet" to simulate a NBD-2 count distribution (parameters n and q = 1/(1+a), because p+q=1 and np/q=at from Figs.2 and 7).
3. The analysis seems to indicate that, if you generate a NBD-1 arrival separation distribution, then this should give you a NBD-2 count distribution, similar to the fact that; if you generate a NBD-0 arrival separation distribution, then this should give you a BD count distribution; and if you generate a gamma arrival time distribution, then this should give you a PD count distribution.

4. Since a Gamma distribution (parameters n and a, see Fig.10) may be used to approximate a NBD-1 arrival separation distribution (parameters n and a = q/p), maybe a Gamma distribution can be used in the time domain to obtain an approximate NBD-2 count distribution. It should be noted that the shape of the cumulative probability curves in WP-14-15 seems to suggest that a Pareto-like, rather than a Gamma distribution, will give a better fit to the data, at least in the time domain.
5. With regard to the discrete interarrival separation distributions, I don't see how a single one-parameter (p) geometric distribution (GD) could generate a two-parameter (n and p) NBD or BD. However, it would seem that a set of n GD-1 or n GD-0 interarrival separation distributions, could be used to generate NBD-1 or NBD-0 arrival separation distributions, respectively. In other words, to generate a NBD-1 arrival separation distribution, use n fruit generators, each set with modified geometric (GD-1) interarrival separation distributions. This should result in a NBD-2 or BD count distribution, per item 3 above.
6. To generate a modified geometric (GD-1) interarrival separation distribution, it seems to me that it is necessary to pick a bin size wide enough so that after the first 20-microsecond ATCRBS (Mode C and 15-pulse Mode 3/A) reply, a second 20-microsecond ATCRBS reply will be generated in the same time bin with a probability of p. For a uniform probability of generating an ATCRBS reply in a time bin, this would seem to require the bin size T = 40/p microseconds, where the NBD-1 arrival separation distribution you are trying to generate is NB(x;n,p) given in Fig.10.

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Arrival Process Distributions

Process	Arrival Count	Interarrival Separation	Arrival Separation
Bernoulli (F-D)	Binomial	Geometric-0	Negative Binomial-0
Poisson (M-B)	Poisson	Exponential	Gamma
Pascal (B-E)	Negative Binomial-2	Geometric-1	Negative Binomial-1

Figure 1

Arrival Count Distribution

Fruit Count Distribution	x = # Arrivals Prob. f(x)	Parameters q=1-p p=(0,1), n=1,2,…	Mean Variance
Binomial	B(x;n,p) x = 0,1,…,n	n!/x!(n-x)! (p^x)(q^(n-x))	M = np V = npq
Poisson	P(x;M), M=np x = 0,1,2,.	(M^x)/x! exp(-M)	M = np V = np
Negative Binomial-2	NB(x;n,q) x = 0,1,2,…	(n+x-1)!/x!(n-1)! (p^x)(q^n)	M = np/q V = np/q^2

Figure 2

Reception Probability Models I

Probability	f(0)	f(1)	Linear (1-kx) Decoder
Binomial	q^n	np(q^(n-1))	1-knp
Poisson	exp(-M)	M(exp(-M))	1-kM
Negative Binomial-2	q^n	np(q^n)	1-knp/q

Figure 3

Reception Probability Models II

Decode Prob.	Geometric (1-k)^x	Linear (1-kx)	Quadratic (1-k(x^2))
B(x;n,p)	(1-kp)^n	1-knp	1-knp(np+q)
P(x;M)	exp(-kM)	1-kM	1-kM(M+1)
NB(x;n,q)	1/(1+kp/q)^n	1-knp/q	1-k(np/q)((np/q)+(1/q))

Figure 4

Method of Moments Estimates

M=Mean V=Variance	q = 1 - p	n	Quadratic Decode Reception Prob.
B(x;n,p) M>V	V/M	(M^2)/(M-V)	1-k((M^2)+V)
P(x;M) M=V			1-k((M^2)+M)
NB(x;n,q) M<V	M/V	(M^2)/(V-M)	1-k((M^2)+V)

Figure 5

Reception Probability Estimates

Decode Fruit/sec	Linear 'Any'	Quadratic Poisson V/M=1	Quadratic NBD V/M=2
10K App. I LDPU	.96 .92	.98 .96	.97 .94
20K App. I LDPU	.92 .84	.94 .88	.92 .84
30K App. I LDPU	.88 .76	.88 .76	.85 .70

Figure 6

Inter-arrival Separation

f(0;n,M)	f(0) = 1-P(t) M = at	g(t) = dP(t)/dt Continuous	g(x), x=n=t Discrete
p = M/n B(0;n,p) = (1-M/n)^n	(1-at/n)^n	a(1-at/n)^(n-1) (Beta)	a(1-a)^(x-1) x=1,2,… Geometric-0
P(0;M) = exp(-M)	exp(-at)	a(exp(-at)) Exponential	
q=1/(1+M/n) NB(0;n,q) = 1/(1+M/n)^n	1/(1+at/n)^n	a/((1+at/n)^(n+1)) (Pareto)	a/((1+a)^(x+1)) x=0,1,2,… Geometric-1

Figure 7

Interarrival Time

Interarrival Time	Mean	Variance	Variance/ Mean
a = p Geometric-0	1/a	(1-a)/(a^2)	(1-a)/a
Exponential	1/a	1/(a^2)	1/a
a = p/q Geometric-1	1/a	(1+a)/(a^2)	(1+a)/a

Figure 8

Geometric Distribution

g(x)	Distribution	Mean	Variance
GD-0 G0(x;p)	p(q^(x-1)) x=1,2,…	1/p	q/(p^2)
GD-1 G(x;p)	p(q^x) x=0,1,…	q/p	q/(p^2)

Figure 9

Arrival Separation Distribution

Separation Distribution	Parameters q=1-p p=(0,1), n=1,2,…	Mean	Variance
NBD-0 NB0(x;n;p) x=n,n+1,…	(x-1)!/(x-n)!(n-1)! (p^n)(q^(x-n))	n/p	nq/(p^2)
Gamma GA(t;n,a)	(a/(n-1)!) ((at)^(n-1))exp(-at)	n/a	n/(a^2)
NBD-1 NB(x;n,p) x=0,1,2,…	(n+x-1)!/x!(n-1)! (p^n)(q^x)	nq/p	nq/(p^2)

Figure 10